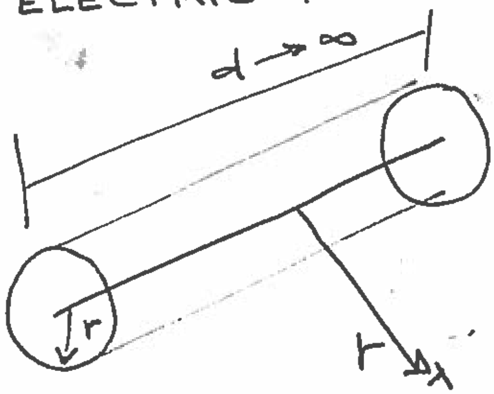
LECTURE 6

09/14/17

ELECTRIC FIELD GENERATED BY A LINE CHARGE



$$\oint \underbrace{\varepsilon \bar{E}}_{\bar{D}}\, d\bar{a} = \int_V q\, dV$$

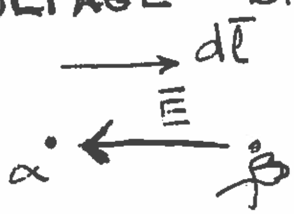
$\bar{E}$ has only radial component

$$2\pi r\, \cancel{d} \cdot \varepsilon \cdot E_r = \overbrace{q\, \cancel{d}}^{Q}$$

q: charge density [C/m]

$$E_r = \frac{q}{2\pi r \varepsilon}$$

VOLTAGE DIFFERENCE



$$V_\beta - V_\alpha = -\int_\alpha^\beta \bar{E}\, d\bar{l}$$

$$\underbrace{-\int_{R_\alpha}^{R_\beta} \bar{E}\, d\bar{l}}$$

$$V_{\beta\alpha} : V_\beta - V_\alpha = -\int_{R_\alpha}^{R_\beta} \frac{q}{2\pi r \varepsilon}\, dr = \frac{q}{2\pi \varepsilon} \ln \frac{R_\alpha}{R_\beta}$$

-Capacitance:

$$Q = C \cdot V$$

Same story as for the inductance

place $-q$ at R_α → and compute the relation between the voltage difference and q

-Per-unit length capacitance

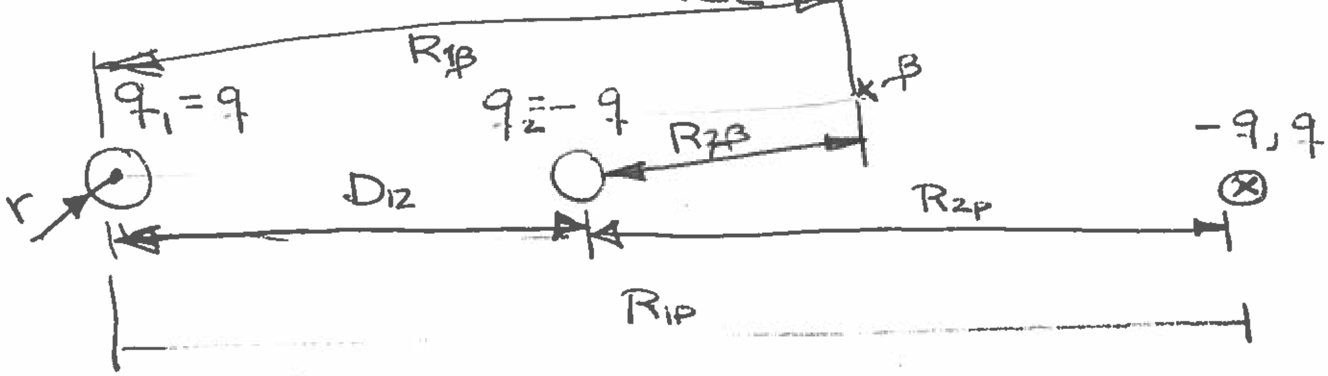
$$q = C \cdot V \longrightarrow C = \left(\frac{1}{2\pi\varepsilon} \ln \frac{R_\alpha}{r'} \right)^{-1}$$

$R_\alpha \to \infty$

$\ln \frac{R_\alpha}{r'} \to \infty \quad C \to 0$

$$C = \frac{2\pi\varepsilon}{\ln \frac{R_\alpha}{r'}}$$

TWO-WIRE CAPACITANCE



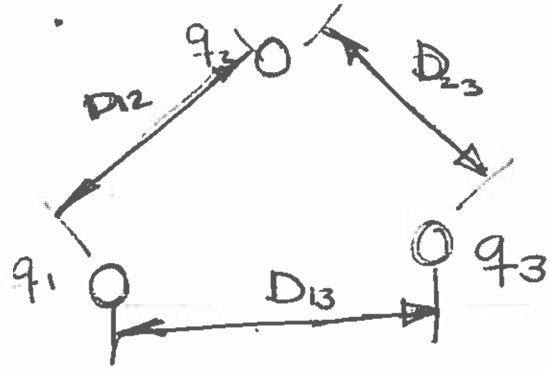
$$V_\beta = \frac{q_1}{2\pi\varepsilon} \operatorname{Ln} \frac{R_{1p}}{R_{1\beta}} + \frac{+q_2}{2\pi} \operatorname{Ln} \frac{R_{2p}}{R_{2\beta}} =$$

$$= \frac{1}{2\pi\varepsilon}\left[\operatorname{Ln} \frac{R_{2\beta}}{R_{1\beta}} + \ln \frac{R_{1p}}{R_{2p}}\right] q$$

Take β to be a point on the surface conductor 1

$$V_1 = \frac{1}{2\pi\varepsilon} \operatorname{Ln} \frac{D_{12}}{r} \cdot q_1 \longrightarrow \boxed{C = \frac{2\pi\varepsilon}{\operatorname{Ln} \frac{D_{12}}{r}}}$$

THREE WIRE CAPACITANCE



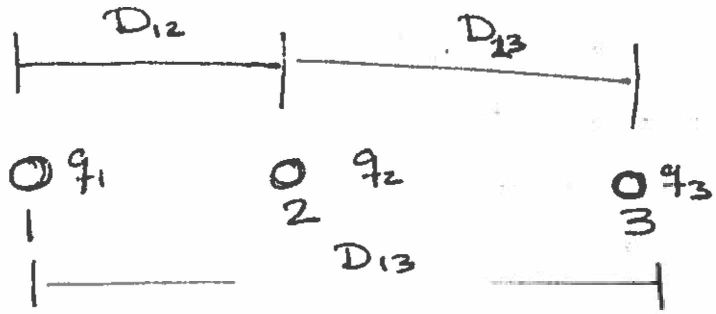
$q_1 + q_2 + q_3 = 0$

$$D_{12} = D_{13} = D_{23}$$

$$q_1 + q_2 + q_3 = 0$$

$$V_1 = \frac{1}{2\pi\varepsilon} \operatorname{Ln} \frac{D}{r} q_1$$

$$\boxed{C = \frac{2\pi\varepsilon}{\operatorname{Ln} \frac{D}{r}}}$$



$$V_1 = \frac{1}{2\pi\varepsilon}\left[\ln\frac{1}{r}q_1 + \ln\frac{1}{D_{12}}q_2 + \ln\frac{1}{D_{13}}q_3\right]$$

As before, by using transposition, we can get to a expression of the form

$$V = \frac{1}{2\pi\varepsilon}\ln\frac{\sqrt[3]{D_{12}D_{23}D_{13}}}{r}q_1$$

$$= \frac{1}{2\pi\varepsilon}\ln\frac{D_m}{r}q_1$$

$$\boxed{C = \frac{2\pi\varepsilon}{\ln\frac{D_m}{r}}, \quad D_m = \sqrt[3]{D_{12}D_{13}D_{23}}}$$

DC RESISTANCE

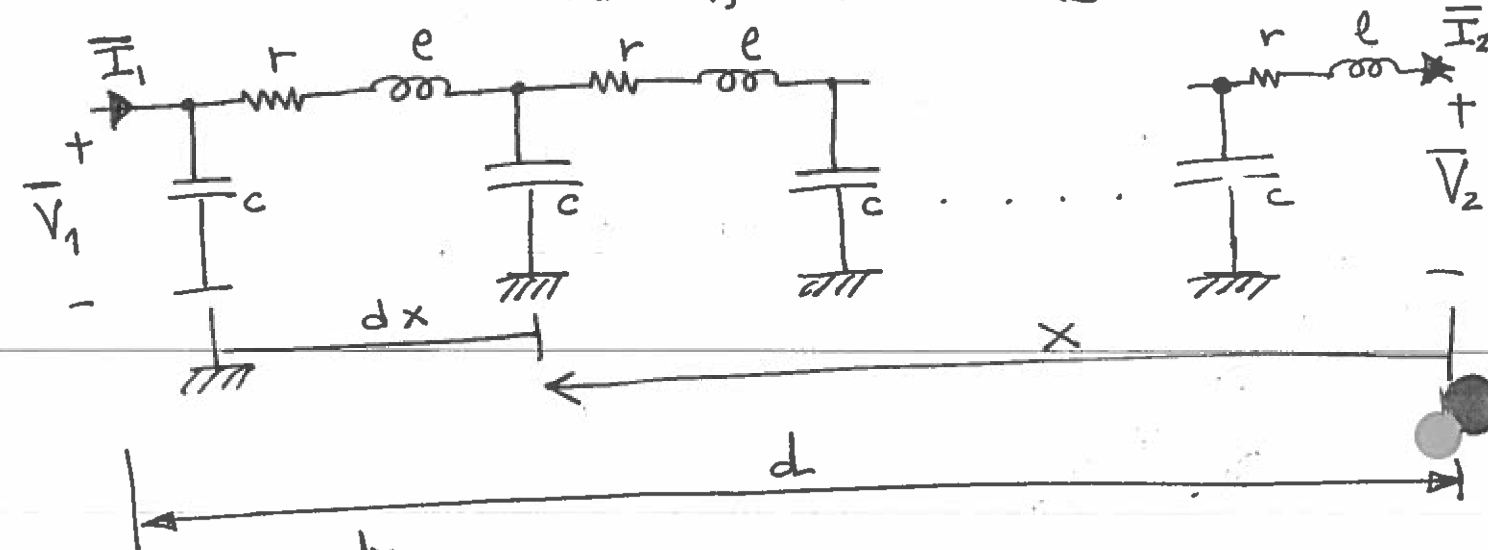
- $r = \frac{\rho}{A}$, ρ is the resistivity of the material and A is the cross-sectional area.

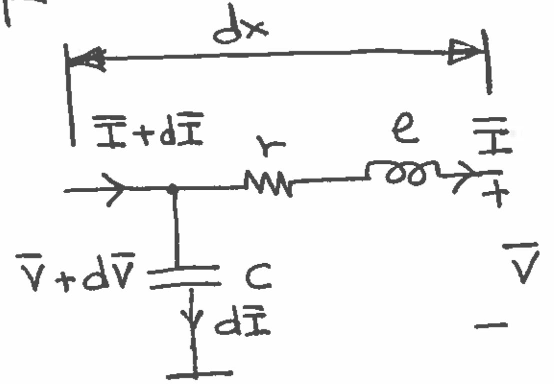
- Because AC current tends to flow towards the exterior surface of the conductor, the resistance @ 60HZ is slightly higher that at DC.

TRANSMISSION LINE MODELING

- DERIVATION OF $\bar V, \bar I$ RELATIONS
- TRANSMISSION MATRIX MODEL
- LUMPED-PARAMETER MODEL

DERIVATION OF TERMINAL $\bar V, \bar I$ RELATIONS





$\bar Z = r + j\omega l$
$\bar Y = j\omega c$

per-unit length
- series impedance
- shunt impedance

Kirchoff's laws:

KVL: $(\cancel{\bar V} + d\bar V) = \cancel{\bar V} + (r + j\omega l)\,dx\,\bar I$

KCL: $\cancel{\bar I} + d\bar I = \cancel{\bar I} + \frac{dx}{(j\omega c)^{-1}}(\bar V + d\bar V) = \bar V \frac{dx}{(j\omega c)^{-1}} + d\bar V \frac{dx}{(j\omega c)^{-1}}$

$\simeq \bar V \frac{dx}{(j\omega c)^{-1}}$

- Define $\gamma := \sqrt{\bar{Z} \cdot \bar{Y}}$ — referred to it as the propagation constant.
- If $r=0 \longrightarrow \gamma = j\sqrt{\ell c}\,\omega = j\omega\sqrt{\ell c}$

The solution is:

$$\begin{bmatrix} \bar{V}(x) \\ \bar{I}(x) \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{e^{\gamma x}+e^{-\gamma x}}{2} & \sqrt{\frac{\bar{Z}}{\bar{Y}}}\,\frac{e^{\gamma x}-e^{-\gamma x}}{2} \\ \sqrt{\frac{\bar{Y}}{\bar{Z}}}\cdot\frac{e^{\gamma x}-e^{-\gamma x}}{2} & \frac{e^{\gamma x}+e^{-\gamma x}}{2} \end{bmatrix}} \begin{bmatrix} \bar{V}_2 \\ \bar{I}_2 \end{bmatrix}$$

$$x = d : \begin{cases} \bar{V}(d) = \bar{V}_2 \\ \bar{I}(d) = \bar{I}_2 \end{cases}$$

$\bar{Z}_c$

$$\begin{bmatrix} \bar{V}_1 \\ \bar{I}_1 \end{bmatrix} = \begin{bmatrix} \frac{e^{\gamma d}+e^{-\gamma d}}{2} & \sqrt{\frac{\bar{Z}}{\bar{Y}}}\,\frac{e^{\gamma d}-e^{-\gamma d}}{2} \\ \sqrt{\frac{\bar{Y}}{\bar{Z}}}\,\frac{e^{\gamma d}-e^{-\gamma d}}{2} & \frac{e^{\gamma d}+e^{-\gamma d}}{2} \end{bmatrix} \begin{bmatrix} \bar{V}_2 \\ \bar{I}_2 \end{bmatrix}$$

$\bar{Z}_c^{-1}$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \bar{V}_2 \\ \bar{I}_2 \end{bmatrix} \longleftarrow \text{TRANMISSION MATRIX MODEL}$$

$$A = \frac{e^{\gamma d}+e^{-\gamma d}}{2} = \cosh \gamma d$$

$$B = \sqrt{\frac{\bar{Z}}{\bar{Y}}}\,\frac{e^{\gamma d}-e^{-\gamma d}}{2} = \bar{Z}_c\,\frac{e^{\gamma d}-e^{-\gamma d}}{2} = \bar{Z}_c \sinh \gamma d$$

- Characteristic impedance: $\bar{Z}_c = \sqrt{\frac{\bar{Z}}{\bar{Y}}}$