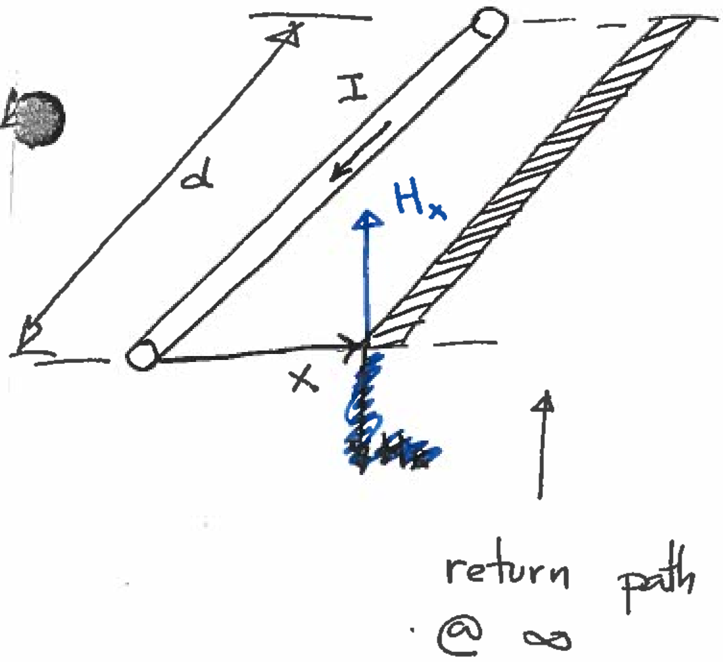
# LECTURE 5

09/12/17

## INDUCTANCE OF A SINGLE WIRE




- We are going to use Ampere's law to first obtain $\ell$

We will start with a single infinitely long wire

- We are going to compute the flux linkages and relate to the current through the wire. Then by linearity, we will ~~be~~ able to to obtain $\ell$ [H/m]

- We will assume that the current density through the wire is uniform and the wire has a radius r.

- We will compute the flux linkages outside the wire first, and then inside the wire.

(i) Flux linkages trought a surface outside the wire, infinitinely long and of width R. (

$$H_x 2\pi X = I \longrightarrow H_x = \frac{I}{2\pi X}$$

$$\varnothing_1 = \int_r^R \frac{\mu_0 I}{2\pi X} \underbrace{d\,dx}_{\text{differential area}} = \frac{\mu_0 d I}{2\pi} \ln \frac{R}{r}$$

$$= \lambda_1$$

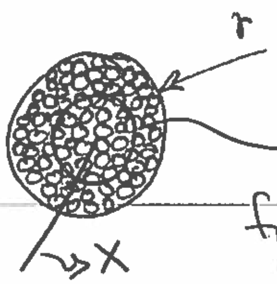
ii) Flux linkage inside the conductor

$$B_2 = \mu \cdot \frac{I_2}{2\pi x} = \frac{\mu I \, x^{2}/r^2}{2\pi \cancel{x}} = \frac{\mu I x}{2\pi r^2}$$

$$d\phi = B\,ds = \frac{\mu I x}{2\pi r^2}\,dx\,d$$

$$\lambda_2 = \int_0^r \underbrace{\frac{\mu I x}{2\pi r^2}\, \overbrace{d \cdot dx}^{\text{diff area}}}_{d\phi} \cdot \frac{x^2}{r^2}$$

" Number of times the flux links the wire




You only link a fraction of them, $x^2/r^2$

$$\text{The } \lambda_2 = \int_0^r \frac{\mu I x^3 d}{2\pi r^4}\,dx = \left.\frac{\mu I x^4 d}{8\pi r^4}\right|_0^r = \frac{\mu I d}{8\pi}$$

Total flux linkages

$$\lambda = \lambda_1 + \lambda_2$$

$$= \frac{\mu_0 d I}{2\pi} \ln \frac{R}{r} + \frac{\mu I d}{8\pi} = \frac{\mu_0 d I}{2\pi}\left(\ln \frac{R}{r} + \frac{\mu_r}{4}\right)$$

$$= \frac{\mu_0 d I}{2\pi}\left(\ln R - \ln r - \ln e^{-\mu_r/4}\right) = \frac{\mu_0 d}{2\pi} \ln \frac{R}{r'}\, I$$

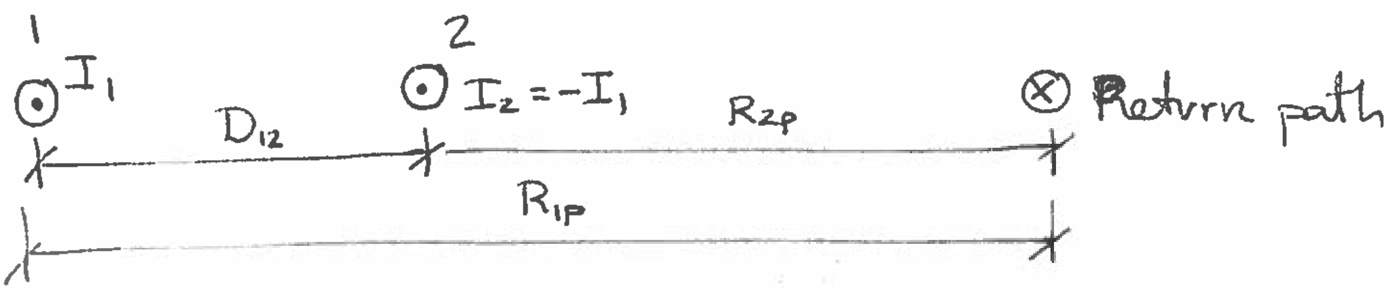
$$\lambda = L \cdot I \qquad L = \frac{\mu_0 d}{2\pi} \ln \frac{R}{r'}$$

$$\boxed{\ell = \frac{L}{d} = \frac{\mu_0}{2\pi} \ln \frac{R}{r'}}$$

← This is not a finite quantity as $R \to \infty$.

- We are going to use superposition now to obtain the inductance of multi-wire arrangements, namely:
  - two-conductors
  - three-conductors

INDUCTANCE OF A TWO-CONDUCTOR LINE




$$\lambda_1 = \lambda_{11} + \lambda_{12}$$

$$\lambda_{11} = \frac{\mu_0 \cdot d}{2\pi} \, Ln \, \frac{R_{1p}}{r'} \cdot I_1 \leftarrow$$ flux linked by conductor 1 due to $I_1$

$$\lambda_{12} = \frac{\mu_0 d}{2\pi} \, Ln \, \frac{R_{2p}}{D_{12}} \, I_2 \leftarrow$$ flux linked by conductor 1 due to $I_2$

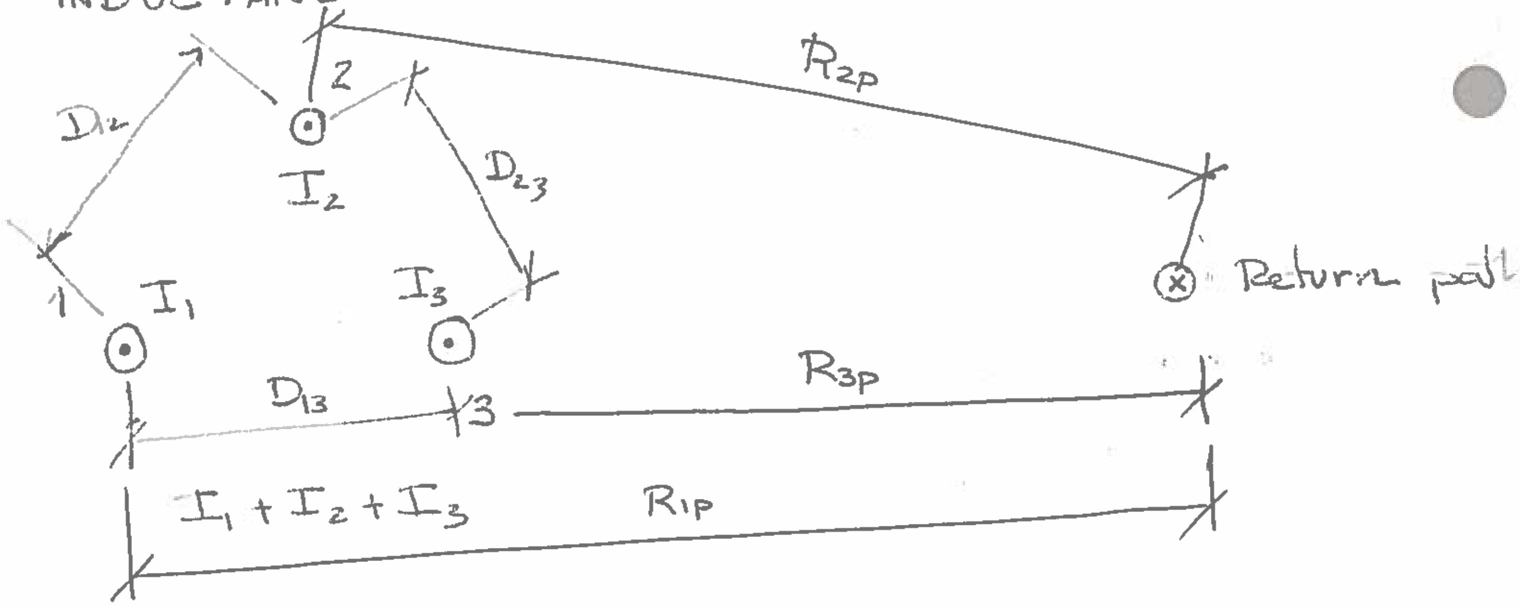
$$\lambda_1 = \frac{\mu_0 d}{2\pi} \left[ Ln \, \frac{R_{1p}}{r'} - Ln \, \frac{\overbrace{R_{1p} - D_{12}}^{\rightarrow R_{2p}}}{D_{12}} \right] I_1$$

$$= \frac{\mu_0 d}{2\pi} \left[ Ln \, \frac{D_{12}}{r'} + \underbrace{\cancel{Ln \, \frac{R_{1p}}{R_{1p} - D_{12}}}}_{\rightarrow 0 \text{ as } R_{1p} \rightarrow \infty} \right] I_1$$

$$L := \frac{\mu_0 d}{2\pi} \, Ln \, \frac{D_{12}}{r'}$$

$$\boxed{\ell = \frac{\mu_0 d}{2\pi} \, Ln \, \frac{D_{12}}{r'}}$$

INDUCTANCE OF A THREE- CONDUCTOR LINE




$$\lambda_1 = \lambda_{11} + \lambda_{12} + \lambda_{13}$$

$$\lambda_{11} = \frac{\mu_o d}{2\pi} Ln \frac{R_{1p}}{r'} \cdot I_1$$

$$\lambda_{12} = \frac{\mu_o d}{2\pi} Ln \frac{R_{2p}}{D_{12}} I_2$$

$$\lambda_{13} = \frac{\mu_o d}{2\pi} Ln \frac{R_{3p}}{D_{12}} I_3$$

$$\lambda_1 = \frac{\mu_o d}{2\pi}\Big[Ln \frac{1}{r'} I_1 + \overbrace{Ln \frac{1}{D_{12}} I_2 + Ln \frac{1}{D_{13}} I_3}^{\text{Coupling with other phase}}$$

$$+ Ln R_{1p} I_1 + Ln R_{2p} I_2 + Ln R_{3p} I_3\Big]$$

$D = D_{12} = D_{13}$, $I_2 + I_3 = -I_1$, $R_{1p} \simeq R_{2p} \simeq R_{3p} \doteq R_p$

$$= \frac{\mu_o d}{2\pi}\Big[Ln \frac{D}{r'} I_1 + Ln R_p \cdot \underbrace{(I_1 + I_2 + I_3)}_{0}\Big]$$

$$\lambda = \frac{\mu_o d}{2\pi} Ln \frac{D}{r'} I_1$$

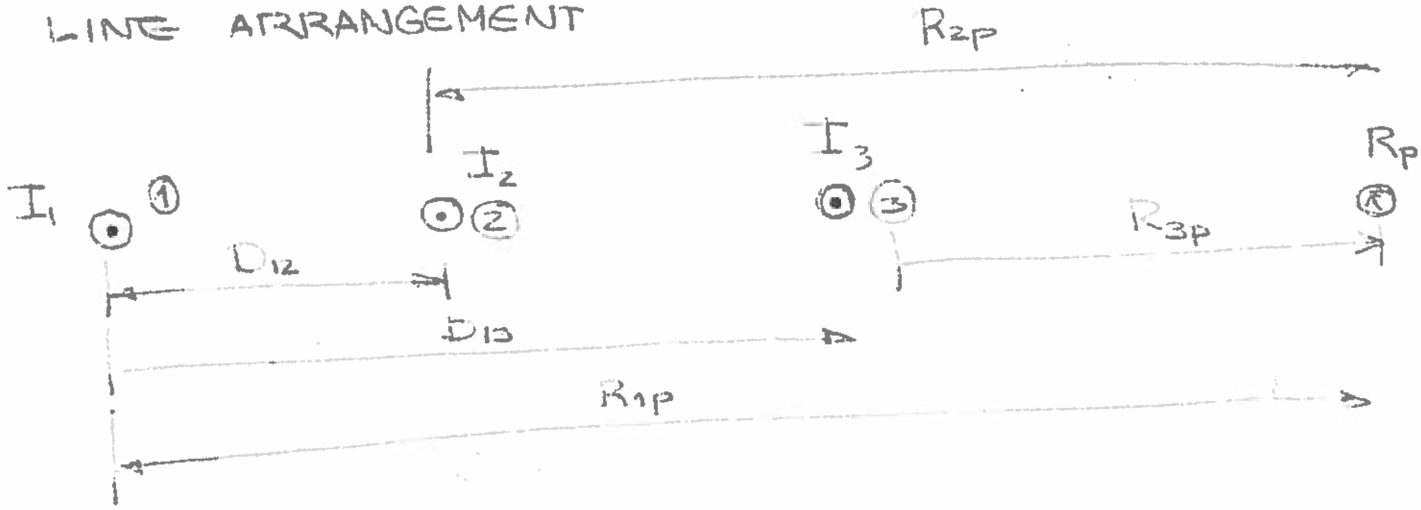
$$\boxed{\ell = \frac{\mu_o}{2\pi} Ln \frac{D}{r'}}$$

$$\lambda_1 = \frac{\mu_0 d}{2\pi} \operatorname{Ln} \frac{D}{r'} I_1$$

$$\boxed{l = \frac{\mu_0}{2\pi} \ln \frac{D}{r'}}$$

Even if there was coupling between the phases, in a symmetric configuration, the magnetic coupling can be modeled using self-inductances: GOOD!

INDUCTANCE OF A THREE CONDUCTOR: LINE ARRANGEMENT




$$\lambda_1 = \lambda_{11} + \lambda_{12} + \lambda_{13}$$

$$\lambda_1 = \frac{\mu_0 d}{2\pi} \operatorname{Ln} \frac{R_{1p}}{r'} I_1$$

$$\lambda_2 = \frac{\mu_0 d}{2\pi} \operatorname{Ln} \frac{R_{2p}}{D_{12}} I_2$$

$$\lambda_3 = \frac{\mu_0 d}{2\pi} \operatorname{Ln} \frac{R_{3p}}{D_{13}} I_3$$

$$\lambda_1 = \frac{\mu_0 d}{2\pi}\left(\ln\frac{R_{1p}}{r'} I_1 + \ln\frac{R_{2p}}{D_{12}} I_2 + \ln\frac{R_{3p}}{D_{13}} I_3\right)$$

$$= \frac{\mu_0 d}{2\pi}\left(\ln\frac{1}{r'} I_1 + \ln\frac{1}{D_{12}} I_2 + \ln\frac{1}{D_{13}} I_3 + I_1 \ln R_{1p} + I_2 \ln R_{2p} + I_3 \ln R_{3p}\right)$$

$$= \frac{\mu_0 d}{2\pi}\left(\ln\frac{1}{r'} I_1 + \ln\frac{1}{D_{12}} I_2 + \ln\frac{1}{D_{13}} I_3\right)$$

$$+ \frac{\mu_0 d}{2\pi}\left(\cancelto{0}{\ln R_p (I_1 + I_2 + I_3)}\right)$$

$$\lambda_1 = \frac{\mu_0 d}{2\pi}\left(\ln\frac{1}{r'} I_1 + \ln\frac{1}{D_{12}} I_2 + \ln\frac{1}{D_{13}} I_3\right)$$

$$\left.\begin{aligned} l_{11} &= \frac{\mu_0}{2\pi}\ln r' \\ l_{12} &= \frac{\mu_0}{2\pi}\ln\frac{1}{D_{12}} \\ l_{13} &= \frac{\mu_0}{2\pi}\ln\frac{1}{D_{13}} \end{aligned}\right\}$$

There is magnetic coupling between the phases!

$$\lambda_1 = \frac{\mu_0 d}{2\pi}\left(\ln\frac{1}{r'} I_1 + \ln\frac{1}{D_{12}} I_2 + \ln\frac{1}{D_{13}} I_3\right)$$

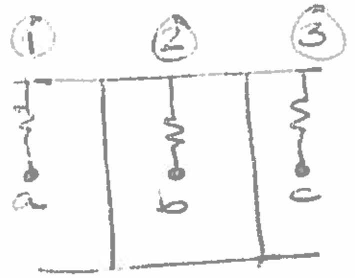
$$\lambda_2 = \frac{\mu_0 d}{2\pi}\left(\ln\frac{1}{r'} I_2 + \ln\frac{1}{D_{12}} I_1 + \ln\frac{1}{D_{23}} I_3\right)$$

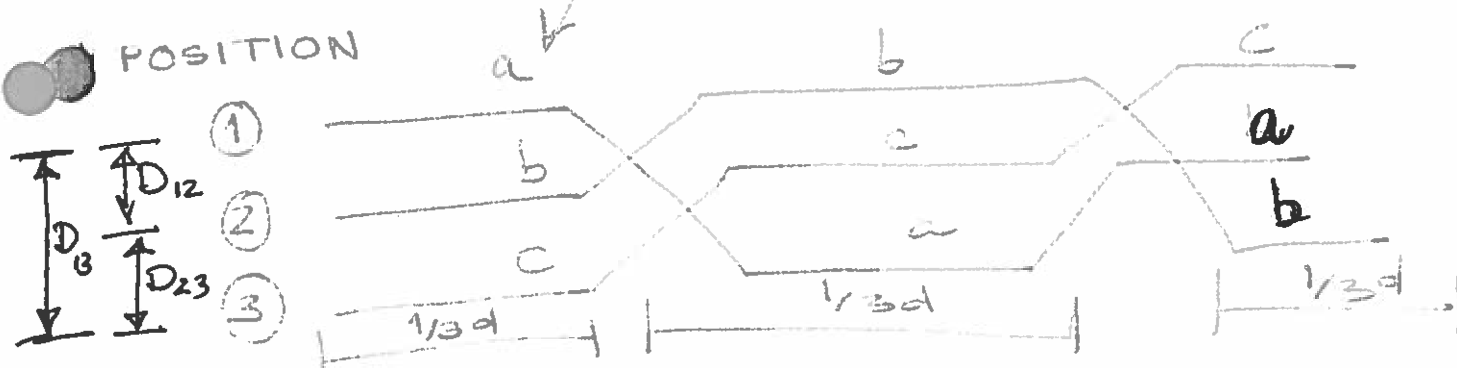
$$\lambda_3 = \frac{\mu_0 d}{2\pi}\left(\ln\frac{1}{r'} I_3 + \ln\frac{1}{D_{23}} I_2 + \ln\frac{1}{D_{13}} I_3\right)$$

$$I_1 + I_2 + I_3 = 0$$

## TRANSPOSITION

To keep the system balanced over the length of a transmission line, the conductors are rotated so each phase occupies each position of the tower for an equal distance. This is known as <u>TRANSPOSITION</u>




Flux linkages:

$$\lambda_a = \frac{1}{3}\frac{\mu_0 d}{2\pi}\left[I_a \ln\frac{1}{r'} + I_b \cdot \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{13}}\right]$$

$$+\frac{1}{3}\frac{\mu_0 d}{2\pi}\left[I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{13}} + I_c \ln\frac{1}{D_{23}}\right]$$

$$+\frac{1}{3}\frac{\mu_0 d}{2\pi}\left[I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{D_{12}}\right]$$

$$3 I_a \ln\frac{1}{r'} \qquad 3 \cdot I_b \ln\frac{1}{\sqrt[3]{D_{12} \cdot D_{13} D_{23}}} \qquad 3 I_c \ln\frac{1}{\sqrt[3]{D_{12} D_{13} D_{23}}}$$

$$\lambda_a = \frac{\mu_0 d}{2\pi}\left[I_a \ln\frac{1}{r'} + \ln\frac{1}{\sqrt[3]{D_{12} D_{13} D_{23}}}\underbrace{(I_b + I_c)}_{-I_a}\right]$$

$$\lambda_a = \frac{\mu_0 d}{2\pi} \ln \frac{\sqrt[3]{D_{12} D_{13} D_{23}}}{r'} I_a$$

$$l_a = \frac{\mu_0}{2\pi} \ln \frac{D_m}{r'},$$

$D_m = \sqrt{D_{12} D_{13} D_{23}}$ is the geometric mean distance between physical positions.

GAUSS' LAW

- Similar to Ampere's law for magnetic fields, Gauss' law allows us to compute ~~electrt~~ electric fields created by charges.

- In integral from

$$\oint_S \underbrace{\varepsilon \cdot \bar{E}}_{\bar{D}} d\bar{a} = Q$$

$\bar{E}$ : Electric field intensity [V/m]

$\bar{D}$ : Electric flux density [C/m²]

$\varepsilon$ : Permittivity [F/m]

Q : Charge enclosed by the closed surface S [C]