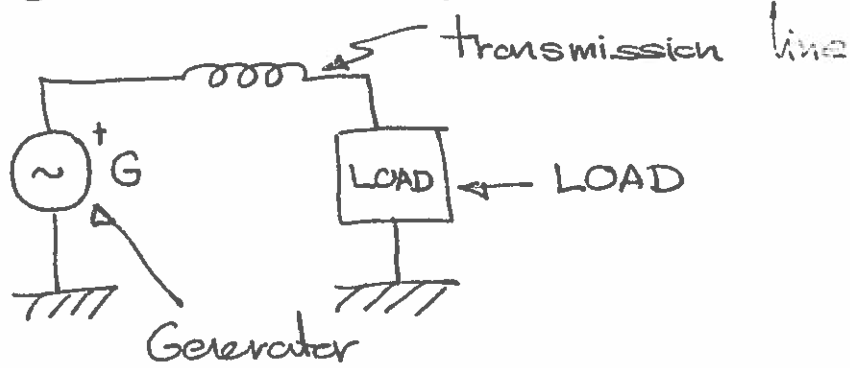
LECTURE 4

09/07/17

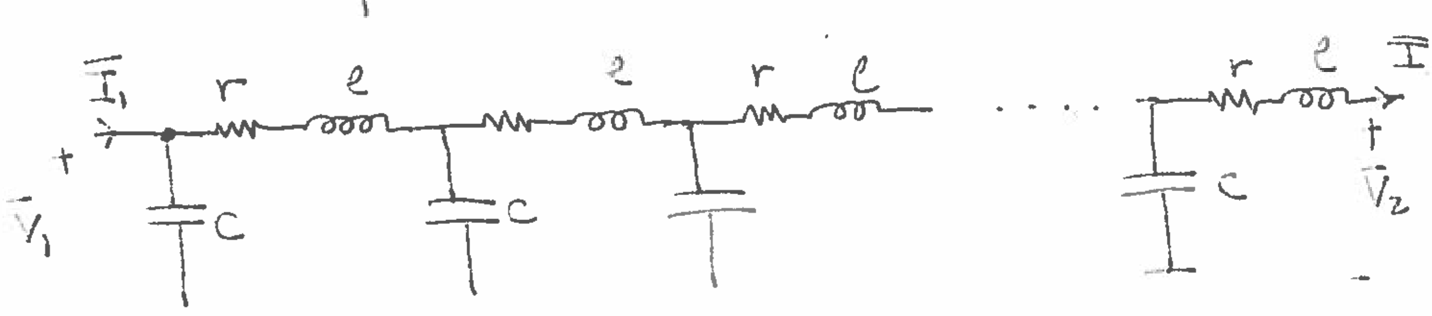
TRANSMISSION LINE MODELING

- Recall the simplest power system model that we discussed in the second lecture.



- In the next few lectures we will develop models for each of these components. [So far we have assume components are linear for the most part. In reality, things are a bit more complicated.]

- The simplest model for a transmission line is an inductor, and indeed we will be using this simplified model a lot, but how do we get there?



- r, l, and c are the per-unit length ~~in~~ resistance, inductance, and capacitance of the transmission line.

Over the next two lectures, we will show how to compute r, l, and c from the geometry of the transmission line. The "hammers" that we will use are Maxwell's equations. In particular, we will use: (i) Ampere's law, (ii) Faraday's law, and (iii) Gauss law.

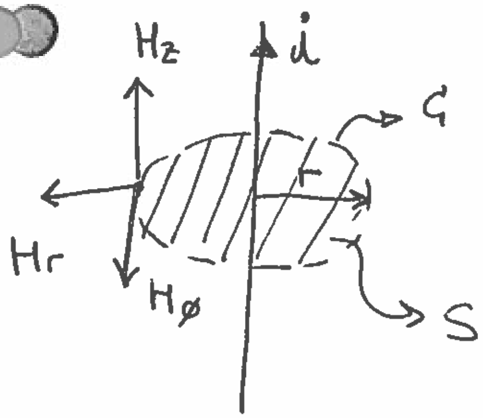
- The first two will help us computing l
- The third one will help us computing c

Ampere's law:

$$\oint_c \bar{H}\, d\bar{l} = \int_s \bar{J}_f \cdot \overbrace{\bar{n}\, da}^{d\bar{a}} + \cancel{\frac{d}{dt}\int_s \varepsilon E\, da}$$

- H: Magnetic field intensity [A-turn/meter]
- dl: differential path vector
- $\oint_c$: line integral around the closed path c ($d\bar{l}$ is tangent to the path)
- $\bar{J}_f$: Current density [A/m²]
- $\bar{n}$: normal vector to S
- S: Surface enclosed by c
- da: differential surface area

MAGNETIC FIELD FROM A SINGLE WIRE



- It is easy to see that H_r and H_z are zero by symmetry.
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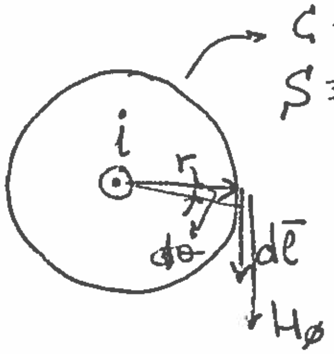


C ≡ circle
S ≡ flat surface enclosed by C

$$\oint_c \bar{H}\, d\bar{l} = \underbrace{\int_s \bar{J}_a \overbrace{\bar{n}\, da}^{d\bar{a}}}_{i}$$

$$dl = r\, d\theta$$

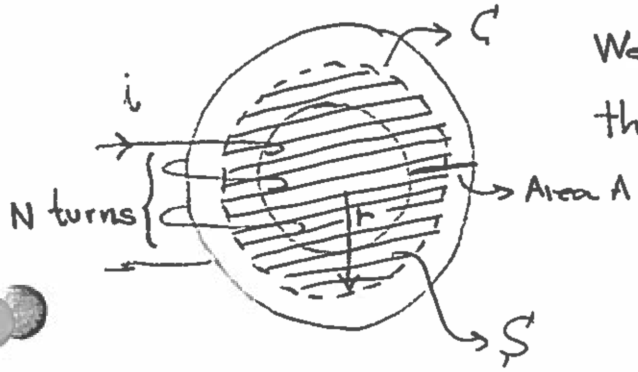
$$\oint_c \bar{H}\, d\bar{l} = \underbrace{\int_c H_\phi\, dl}\ = i$$

$$\int_0^{2\pi} H_\phi\, r\, d\theta = i$$

$$H_\phi \cdot r \cdot \int_0^{2\pi} d\theta = i \longrightarrow 2\pi r H_\phi = i$$

$$\boxed{H_\phi = \frac{i}{2\pi r}}$$

MAGNETIC FIELD INSIDE A MAGNETIC CORE



We assume that $\bar{H}$ is uniform throughout the core.

$$H 2\pi r = N i$$

$$\boxed{H = \frac{N i}{2\pi r}}$$

SOME ADDITIONAL NOTIONS

1. Magnetic flux density (B)

$\boxed{B := \mu H}$, where μ is the so-called permeability

$\mu = \mu_0 \cdot \mu_r$

- μ_0 : permeability of free space : $4\pi 10^{-7}$ H/m
- μ_r : relative permeability :
  - 1 for air
  - 1,000 for magnetic materials.

B is measured in

Tesla [T] or Gauss [G] ; $1T = 10^4 G$

2. Flux (ϕ) : Total magnetic flux passing through a surface S

$$\boxed{\phi = \int_S \bar{B}\, d\bar{a}},$$

$d\bar{a}$ is the differential surface vector, and has a direction normal to the surface.

If $\bar{B}$ is uniform, then we have that

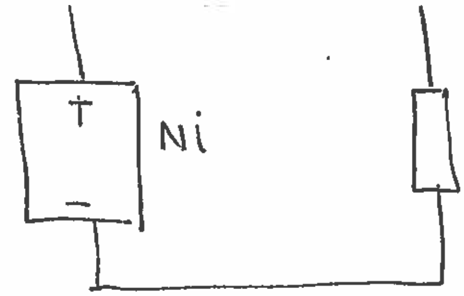
$\boxed{\phi = BA}$, A is the area of the surface.

3. Reluctance ($\mathcal{R}$)

In the coil example, multiply by A on both sides and also by μ :

$$\underbrace{A\cdot\overbrace{\mu H}^{B}}_{\phi} = \mu A \cdot \frac{Ni}{2\pi r} = \frac{Ni}{\underbrace{\frac{\overbrace{\ell}^{\text{length}}}{\mu A}}_{\mathcal{R}}}$$

$$\boxed{\phi = \frac{Ni}{\mathcal{R}}}$$



$\mathcal{R} = \frac{\ell}{\mu A}$

There is a clear analogy between electric & magnetic circuits:

$V \longrightarrow Ni$ (magnetomotive force)

$R \longrightarrow \mathcal{R}$ (reluctance)

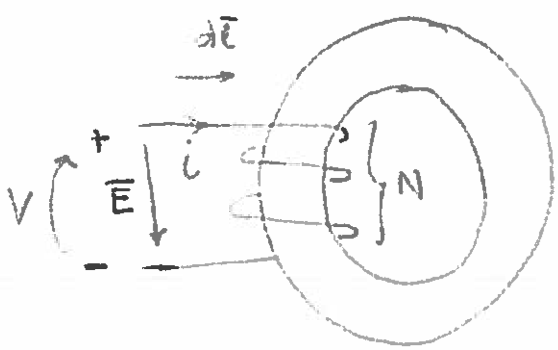
$i \longrightarrow \phi$ (flux)

FARADAY'S LAW

$$\oint_c \bar{E}\, d\bar{\ell} = -\frac{d}{dt}\int_s \bar{B}\, d\bar{a}$$

$$V_+ - V_- = -\int_-^+ \bar{E}\, d\bar{\ell}$$

$d\bar{\ell}$ and $d\bar{a}$ has to be consistent with the right hand rule.



$$-\oint \bar{E}\, d\bar{\ell} = V$$

$$V = \frac{d}{dt}\underbrace{\int_s \bar{B}\, d\bar{a}}_{\lambda}$$

$$\phi = \frac{Ni}{\underbrace{\frac{\ell}{\mu A}}_{\mathcal{R}\ \text{(reluctance)}}}$$

Flux linkages: # of times that the magnetic flux links the current

$$\lambda = N\phi \qquad V = \frac{d\lambda}{dt}$$

$$\lambda = N\phi = \frac{N^2 i}{\frac{\ell}{\mu A}} = \underbrace{\frac{N^2}{\frac{\ell}{\mu A}}}_{L} i$$

$$\boxed{L := \frac{N^2}{\frac{\ell}{\mu A}}} \quad \text{INDUCTANCE} \qquad \frac{d\lambda}{dt} = V = L\frac{di}{dt}$$

Significance: In a linear circuit, the inductance relates the current and the flux linkages. L [H]