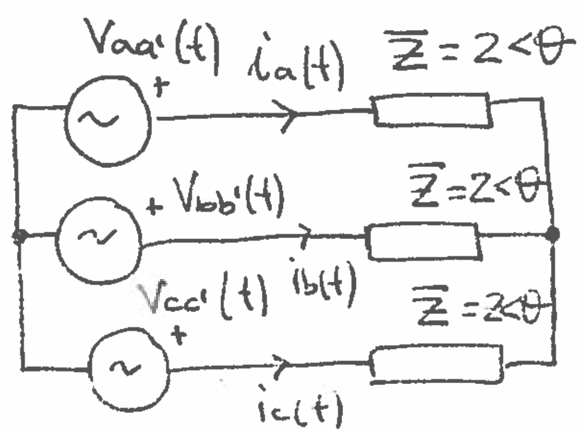
09/05/17

LECTURE 3

- POWER IN THREE-PHASE SYSTEMS
- ANALYSIS OF Y-Δ CONNECTIONS
- OTHER TYPES OF CONNECTIONS
- PER-PHASE EQUIVALENTS (Y-Δ TRANSFORMATION)
- EXAMPLES

POWER IN 3Ø SYSTEMS

We saw that in balanced three-phase systems, all triplets form a balance three-phase system.



$$V_{aa'}(t) = \sqrt{2}V\cos(\omega t)$$

$$V_{bb'}(t) = \sqrt{2}V\cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$V_{cc'}(t) = \sqrt{2}V\cos\left(\omega t + \frac{2\pi}{3}\right)$$

$$i_a(t) = \sqrt{2}I\cos(\omega t - \theta)$$

$$i_b(t) = \sqrt{2}I\cos\left(\omega t - \frac{2\pi}{3} - \theta\right)$$

$$i_c(t) = \sqrt{2}I\cos\left(\omega t + \frac{2\pi}{3} - \theta\right)$$

$$P_a(t) = VI\left[\cos(\theta) + \cos(2\omega t - \theta)\right]$$

$$P_b(t) = VI\left[\cos(\theta) + \cos\left(2\omega t - \theta - \frac{4\pi}{3}\right)\right]$$

$$P_c(t) = VI\left[\cos(\theta) + \cos\left(2\omega t - \theta + \frac{4\pi}{3}\right)\right]$$

$$P_{3\phi}(t) = P_a(t) + P_b(t) + P_c(t)$$

$$= 3VI\cos(\theta) + 0$$

$$\boxed{P_{3\phi}(t) = 3VI\cos(\theta) = P}$$ ← instantaneous power is constant

~~[illegible] POWER~~

Recall that here V and I are phase voltage and current ($V \equiv V_\phi$, $I \equiv I_\phi$)

$$V_L = \sqrt{3}V_\phi, \quad I_L = I_\phi$$

$$\boxed{P_{3\phi} = \sqrt{3}\, V_L \cdot I_L \cdot \cos(\theta)}$$ ← This formula is universal and work for any connection (Y or Δ)

COMPLEX POWER

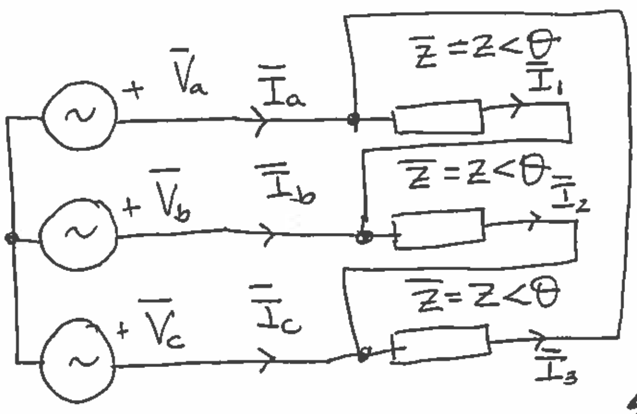
$$\vec{S}_{3\phi} = \vec{S}_a + \vec{S}_b + \vec{S}_c$$

$$= 3VI \cdot [\cos\theta + j\sin\theta)$$

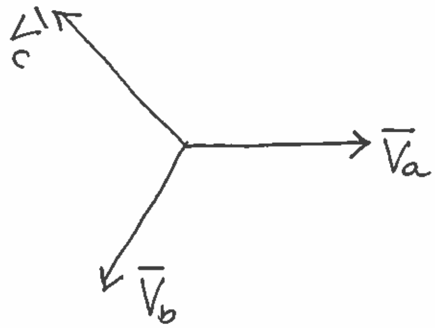
$$= \sqrt{3}\, V_L \cdot I_L \cdot [\cos\theta + j\sin\theta]$$

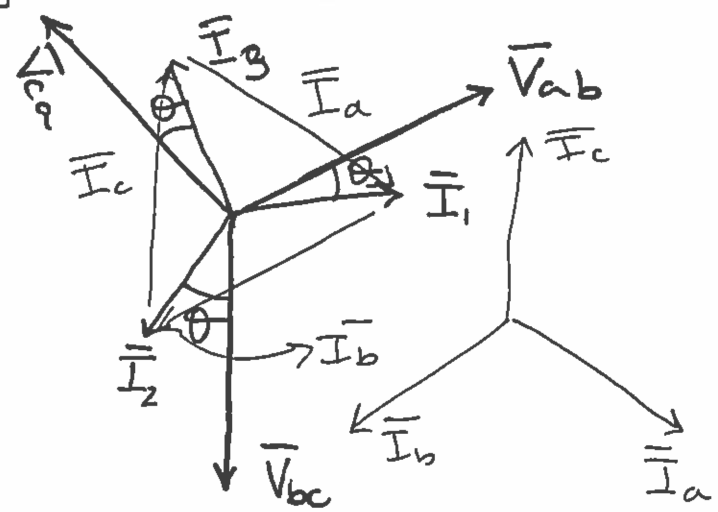
$$\left.\begin{aligned} P &= \sqrt{3}\, V_L \cdot I_L \cdot \cos(\theta) \\ Q &= \sqrt{3}\, V_L \cdot I_L \sin(\theta) \end{aligned}\right\}$$ Universal formulas.

ANALYSIS OF Y-Δ CONNECTION



$0 \le \theta \le \frac{\pi}{2}$





abc ⊕ seq V

$$\bar{I}_a = \bar{I}_1 - \bar{I}_3$$
$$\bar{I}_b = \bar{I}_2 - \bar{I}_1$$
$$\bar{I}_c = \bar{I}_3 - \bar{I}_2$$

3∅ power (Δ)

$$P_{3\phi} = 3 \cdot V_{ab} \cdot I_1 \cos\theta$$
$$= 3\, V_{ab} \cdot \frac{I_a}{\sqrt{3}} \cos\theta$$
$$Q_{3\phi} = 3\ V_{ab}\ I_1 \cdot \sin\theta$$
$$= 3\, V_{ab} \cdot \frac{I_a}{\sqrt{3}} \sin\theta$$

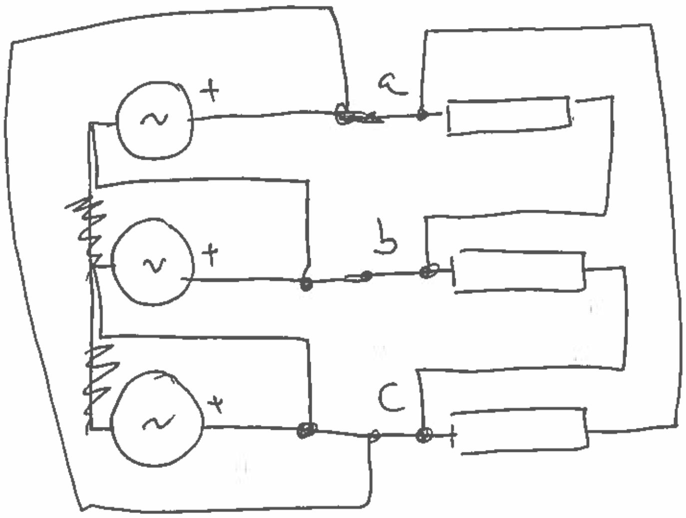
$V_{ab} \equiv I_L$

$$I_1 = \frac{I_a}{\sqrt{3}} = \frac{I_L}{\sqrt{3}}$$

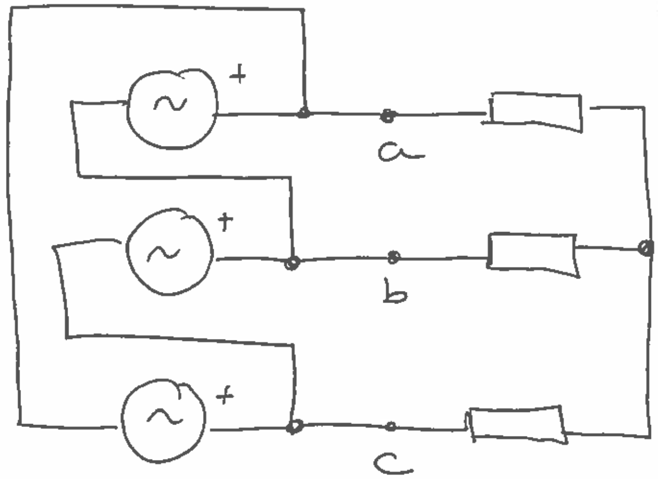
$$\boxed{\begin{aligned} P_{3\phi} &= \sqrt{3}\, V_L \cdot I_L \cdot \cos\theta \\ Q_{3\phi} &= \sqrt{3}\ V_L \cdot I_L \sin\theta \end{aligned}}$$

← same as the Y connection.

OTHER TYPES OF CONNECTIONS



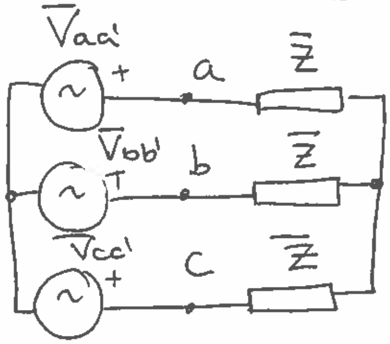
Δ – Δ CONN.



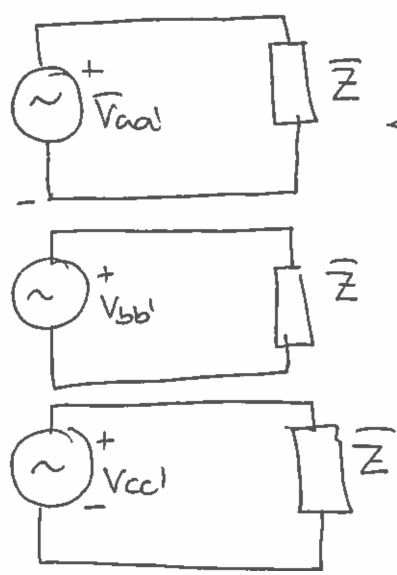
Δ – Y CONN.

PER-PHASE EQUIVALENTS

We have shown that

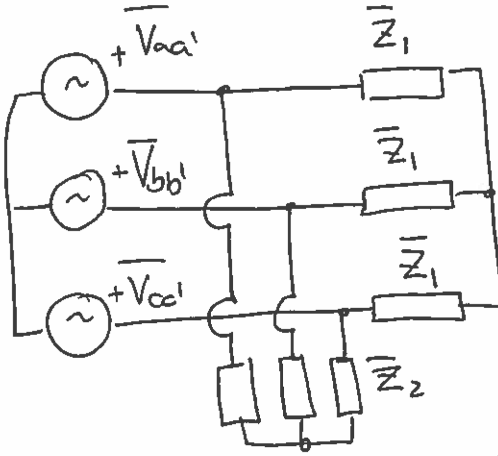


$\equiv$

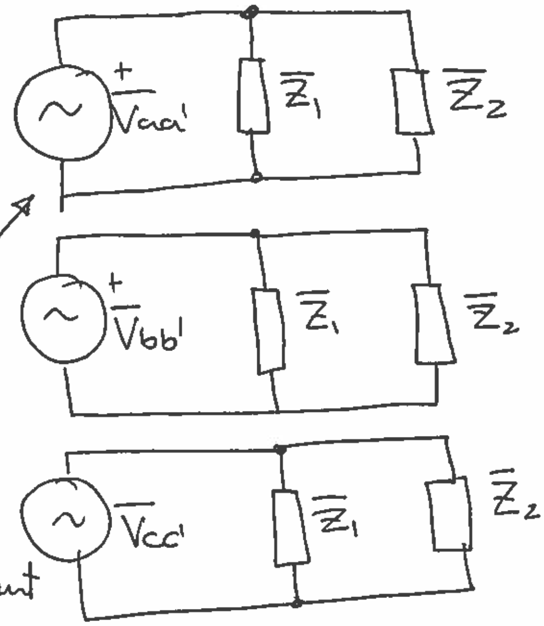


per-phase equivalent

What about ?

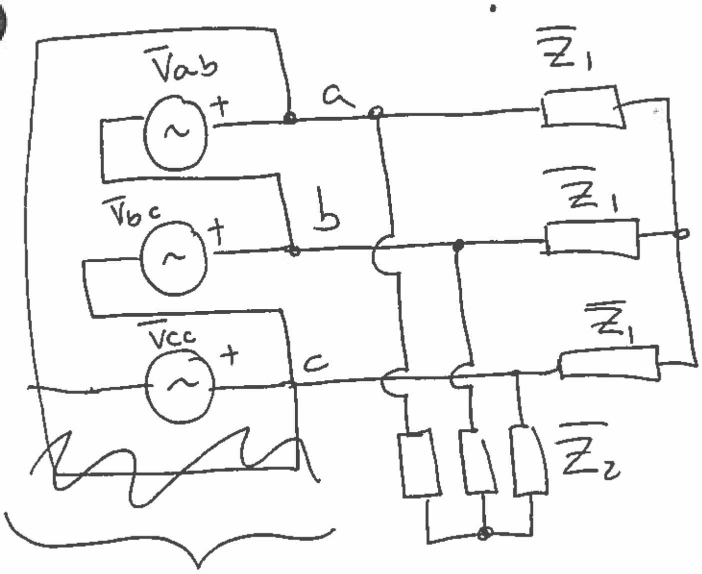


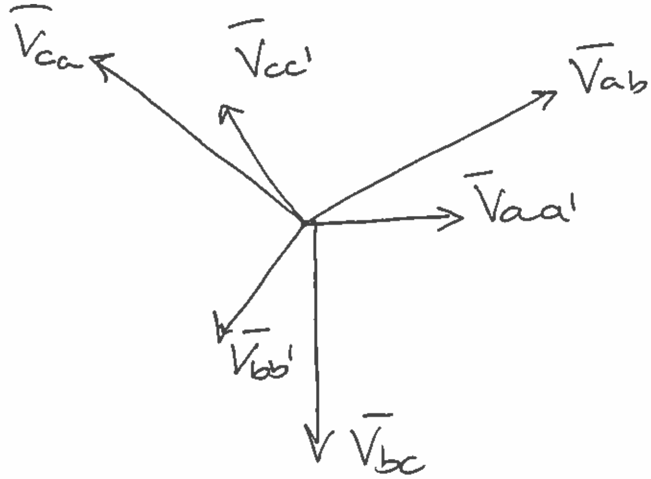
$\equiv$

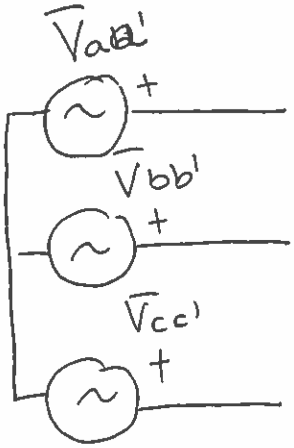


per-phase equivalent

What about ?

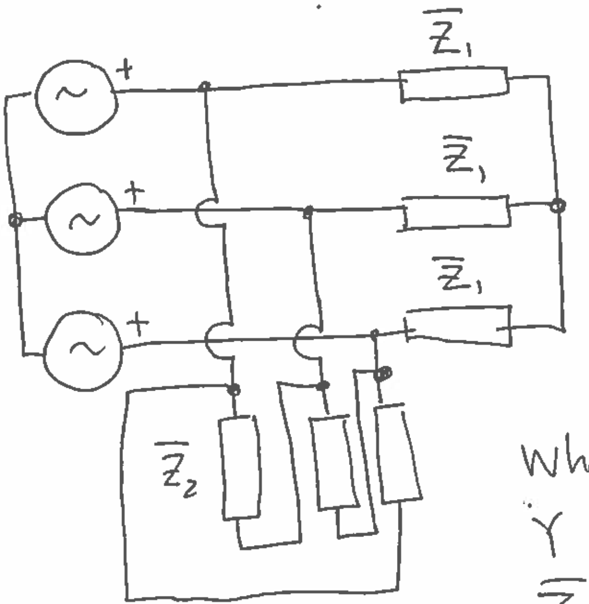




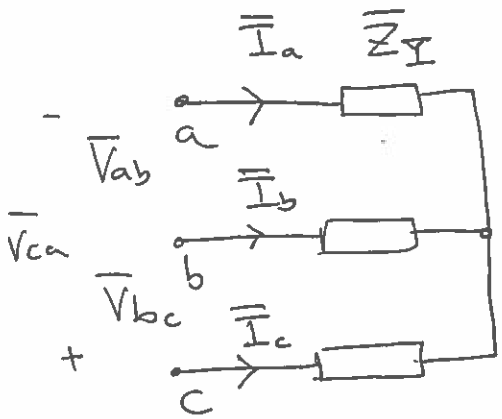


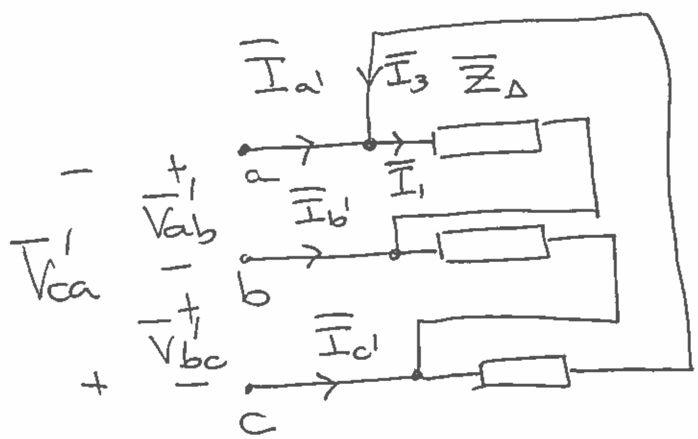
. . . $\rightarrow$ The rest is the same as above.

What about?



What we somehow find a Y-equivalent of the $\bar{Z}_2$ load? If so, we are back to case 1.
[We sort of did that already for the Δ-connected voltage source.]





What is the relation between $\bar{Z}_Y$ and $\bar{Z}_\Delta$ so that

$$\left.\begin{aligned} \bar{V}_{ab} &= \bar{V}'_{ab} \\ \bar{V}_{bc} &= \bar{V}'_{bc} \\ \bar{V}_{ca} &= \bar{V}'_{ca} \end{aligned}\right| \Rightarrow \begin{aligned} \bar{I}_a &= \bar{I}'_a \\ \bar{I}_b &= \bar{I}'_b \\ \bar{I}_c &= \bar{I}'_c \end{aligned}$$

$$\bar{Z}_Y = \frac{1}{3}\bar{Z}_\Delta$$

Δ - connection

$$\bar I_a' = \bar I_1 - \bar I_3$$

$$= \frac{\bar V_{ab}}{\bar Z_\Delta} - \frac{\bar V_{ca}}{\bar Z_\Delta} = \frac{\bar V_{ab}' - \bar V_{ca}'}{\bar Z_\Delta}$$

Y connection

$$\bar V_{ab} = \bar I_a \cdot \bar Z_Y - \bar I_b \cdot \bar Z_Y$$

$$\bar V_{ba} = \bar I_b \bar Z_Y - \bar I_a \bar Z_Y$$

$$\frac{\bar V_{ab} - \bar V_{bc}}{\bar Z_Y} = \bar I_a \overbrace{- \bar I_b - \bar I_c}^{\bar I_a} + \bar I_a = 3 \bar I_a$$

$$\left.\begin{aligned} \bar V_{ab} - \bar V_{bc} &= 3 \bar Z_Y \cdot \bar I_a \\ &\text{vs} \\ \bar V_{ab}' - \bar V_{ca}' &= \bar Z_\Delta \cdot \bar I_a \end{aligned}\right\} \rightarrow \boxed{\bar Z_\Delta = 3 \cdot \bar Z_Y}$$

$$\bar Z_Y = \frac{1}{3} \bar Z_\Delta$$

PROBLEM

LOAD #1: Y-CONNECTED LOAD, 100KVA (3Ø), 0.9 PF lag

LOAD #2: Y-CONNECTED LOAD, 60KW (3Ø), 0.7 PF lead

LOAD #3: Δ-CONNECTED LOAD, 75A (phase current) 0.9 PF lag

a) Total complex power (3ϕ)

LOAD 1: $\bar{S}_1 = 100 \cdot (0.9 + j \sin(\cos^{-1}(0.9)))$
$= 90 + j43.6$

LOAD 2: $\bar{S}_2 = 60 - j61.21$

LOAD 3:

$I_\phi = 75A$

$I_L = \sqrt{3} \cdot 75$
$= 129.9\ A$

$S_3 = \sqrt{3} \cdot V_L \cdot I_L$
$= \sqrt{3} \cdot 480 \cdot 129.9$
$= 108\ KVA$

$\bar{S}_3 = 108\,(0.9 + j \sin(\cos^{-1}(0.9)))$
$= 97.2 + j47.06$

$$\bar{S}_T = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = (90 + 60 + 97.2) + j(43.6 - 61.21 + 47.06)$$
$$= 247.2 + j29.45$$

$S_T = 248.94\ KVA$

b) $I_L = \dfrac{S_T}{\sqrt{3} V_L} = \dfrac{248.94 \cdot 10^3}{\sqrt{3} \cdot 480} = 299.43A$
$\simeq 300A$

c) $Q_T = 29.45 \longrightarrow$ Need to add
$Q_c = -29.45 \longrightarrow$
$Q_c/3 = -9.82\ KVAr$
to be added per phase

d) $I_L' = \dfrac{S_T'}{\sqrt{3} V_L} = \dfrac{247.2 \cdot 10^3}{\sqrt{3} \cdot 480} = 297.33\ A$