

# LECTURE 10

09/28/17

## PER-UNIT THREE-PHASE QUANTITIES

Three-phase quantities may also be normalized by picking appropriate 3 $\phi$  bases. Let  $S_B, V_B, I_B, Z_B$  be the per-phase quantities discussed earlier.

In a natural way, we can define

$$S_B^{3\phi} = 3S_B$$

$$V_B^{l-l} = \sqrt{3} V_B$$

Then

$$S_{pu}^{3\phi} = \frac{S^{3\phi}}{S_B^{3\phi}} = \frac{3S}{3S_B} = S_{pu}$$

↑  
As before

$$V_{pu}^{l-l} = \frac{V^{l-l}}{V_B^{l-l}} = \frac{\sqrt{3} \cdot V}{\sqrt{3} V_B} = V_{pu}$$

Thus, numerically the distinction between per-phase and 3 $\phi$  quantities expressed in per unit may be unimportant

$$Z_B = \frac{V_B^2}{S_B} = \frac{(\sqrt{3} V_B)^2}{3 S_B} = \frac{V_B^{3\phi}}{S_B^{3\phi}}$$

## PER-UNIT ANALYSIS

[S1.] Pick a voltampere base for the whole system.

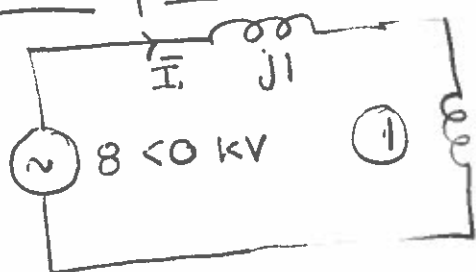
[S2.] Pick one base voltage arbitrarily. Relate all the others by the ratio of the magnitudes of the open-circuit line voltages of each transformer (turn ratio)

[53.] Find the impedance bases in the different sections and express all impedances in consistent per unit terms.

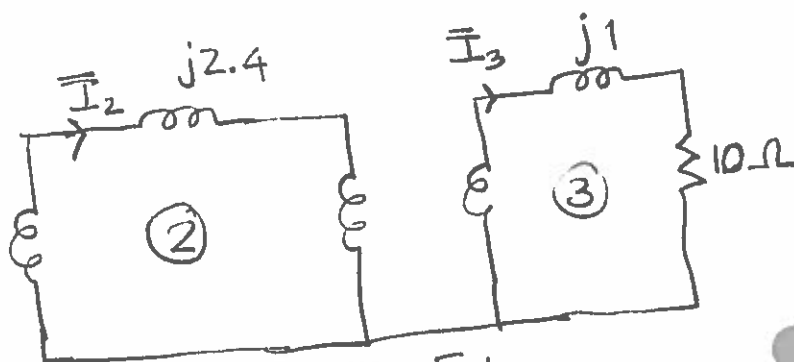
[54.] Draw the impedance diagrams for the entire system, and solve for the desired per unit quantities.

[55.] Convert back to actual quantities if needed/desired.

Example



$$a_1 = 1:10$$



$$a_2 = 5:1$$

Pick

$$S_B = 100 \text{ MVA}$$

$$V_{1B} = 8 \text{ kV} \Rightarrow \begin{cases} V_{2B} = 10 \cdot V_{1B} = 80 \text{ kV} \\ V_{3B} = \frac{1}{5} V_{2B} = 16 \text{ kV} \end{cases}$$

$$Z_{1B} = \frac{(8 \cdot 10^3)^2}{100 \cdot 10^6} = 0.64 \Omega$$

$$I_{1B} = \frac{100 \cdot 10^6}{8 \cdot 10^3} =$$

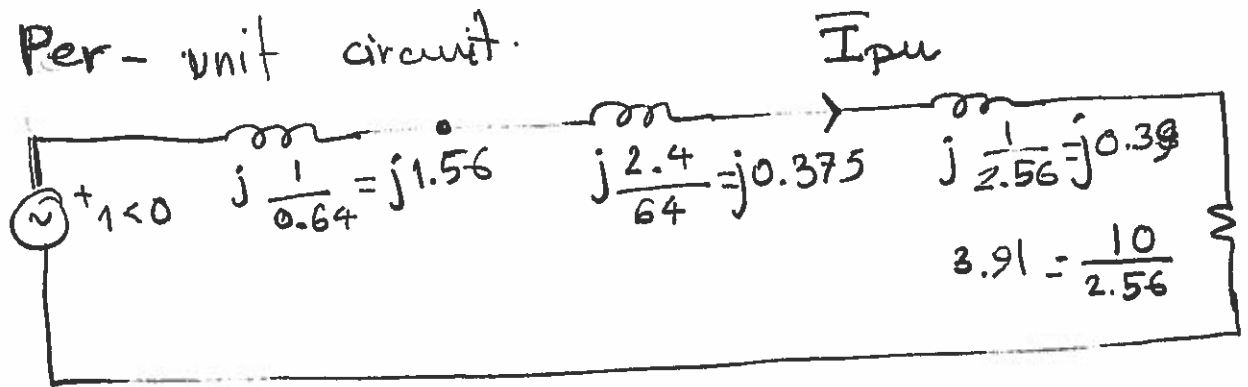
$$Z_{2B} = \frac{(80 \cdot 10^3)^2}{100 \cdot 10^6} = 64 \Omega$$

$$I_{2B} = \frac{100 \cdot 10^6}{80 \cdot 10^3} =$$

$$Z_{3B} = \frac{(16 \cdot 10^3)^2}{100 \cdot 10^6} = 2.56 \Omega$$

$$I_{3B} = \frac{100 \cdot 10^6}{16 \cdot 10^3} =$$

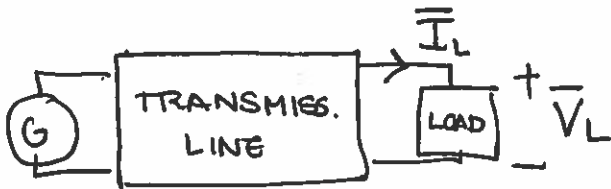
Per-unit circuit.



$$\bar{I}_{pu} = \frac{1 \angle 0}{j(1.56 + 0.375 + j0.39)} = 0.22 \angle -30.8$$

### LOAD MODELS

Recall the simplest power system model below

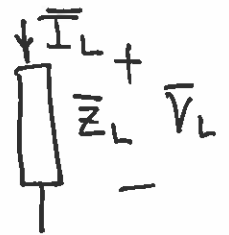


What is an appropriate model for the load?

- We have three different options:

- (i) Constant impedance load
- (ii) Constant current load
- (iii) Constant power load.

(i)  $\bar{Z}_L \rightarrow$  CONSTANT:  $\bar{V}_L = \bar{Z}_L \cdot \bar{I}_L$



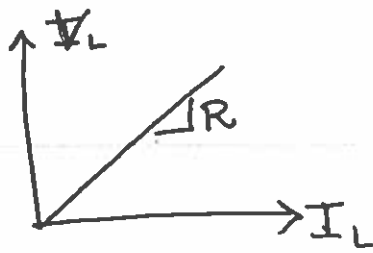
(ii)  $\bar{I}_L \rightarrow$  CONSTANT  $\bar{V}_L$  ← current source.

(iii)  $\bar{V}_L \bar{I}_L^* = P_L + jQ_L$

$\bar{V}_L \bar{I}_L^* = \bar{S}_L \rightarrow$  CONSTANT

If we were dealing

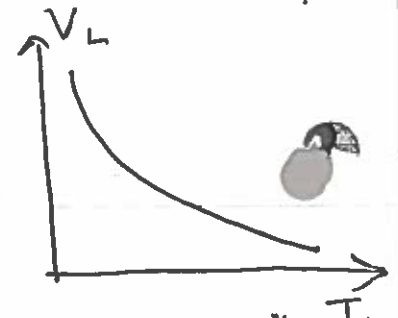
with ~~DC~~  $Q=0$



$$V_L = RI_L$$



$$I_L = \text{CONSTANT}$$



$$P = \overline{V}_L \overline{I}_L^* \\ = V_L I_L$$

- In a real power system, loads (aggregate) are a mix of the three types of loads above, or more precisely, they behave as a mix of (i)-(iii)
- The mix of (i)-(iii) is what is called the ZIP model.
- Types (i) and (ii) are easy to deal with as they exhibit linear behavior. Type (iii) is the most difficult one to deal with due to the nonlinear relation between  $\overline{V}_L$  and  $\overline{I}_L$
- Constant power loads (type (iii)) make the power flow problem formulation (which we will study next) ~~very~~ nonlinear.
- Another reason why it is natural to use constant power load models is due to the fact that forecasting the power demand (as opposed to current or voltage) is relatively easy to do using historical data and weather models.

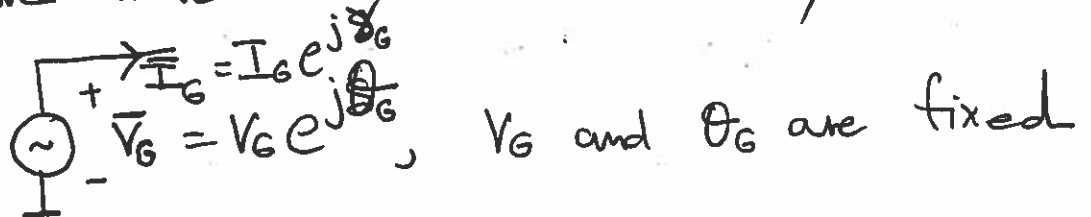
## GENERATOR MODELS

- We mainly deal with two generator models for system-level studies:
  - Static models (
  - Dynamical models (WE WILL TALK LATER) ABOUT THESE
- In ECE 476, we mostly use ~~the~~ static model and a bit of dynamical model at the end.
- You learn more about dynamical models in ECE 431 and ECE 576.

## STATIC MODELS

- (i) Constant voltage source
  - (ii) Constant power source operating with a fixed voltage magnitude.
- (i) Constant voltage source.

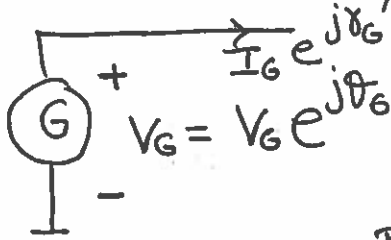
- This is just an ideal voltage source like the ones we have seen in circuit theory



- $V_G$  and  $\theta_G$  are fixed (and known)
- $I_G$  and  $(\theta_G - \delta_G)$  are not known a priori and will be determined by the external system

(ii) Constant power source operating at a fixed voltage magnitude.

- There is no <sup>standard</sup> symbol for this.



$$P_G = V_G I_G \cos(\theta_G - \delta_G)$$

$$Q_G = V_G I_G \sin(\theta_G - \delta_G)$$

- $P_G$  and  $V_G$  are known (fixed)
- $Q_G$  and  $\theta_G - \delta_G$  will be determined by the external system.

## THE POWER FLOW PROBLEM

- The canonical problem in power system analysis.
- In this analysis, the transmission system is modeled by a set of buses (or nodes) interconnected by transmission lines.
- The analysis is appropriate for solving for the system quasi-steady state operating point: voltages and powers.
- The calculation is analogous to the familiar problem of solving for the steady-state voltages and currents in a circuit.