08/31/17

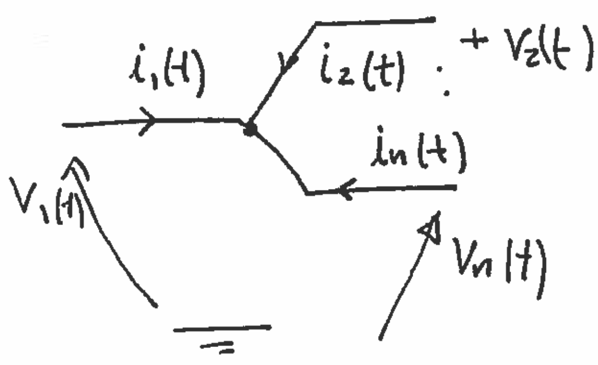
LECTURE 2

- CONSERVATION OF POWER
- REACTIVE POWER COMPENSATION
- THREE-PHASE POWER
  - 3ϕ VOLTAGE SOURCES
  - 3ϕ CONNECTIONS
  - 3ϕ POWER NOTIONS

CONSERVATION OF POWER

- For any node in an electrical network:

(i) Sum of real power into the node must be zero
(ii) Sum of reactive power into the node must be zero



KVL $=$ $\underbrace{\overline{V_i} = \overline{V_j}}_{v_i(t) = v_j(t)}$, $\forall\, i, j$

KCL: $\sum_{j=1}^{n} i_j(t) = 0 \rightarrow$

$$\rightarrow \sum_{j=1}^{n} \overline{I_j} = 0$$

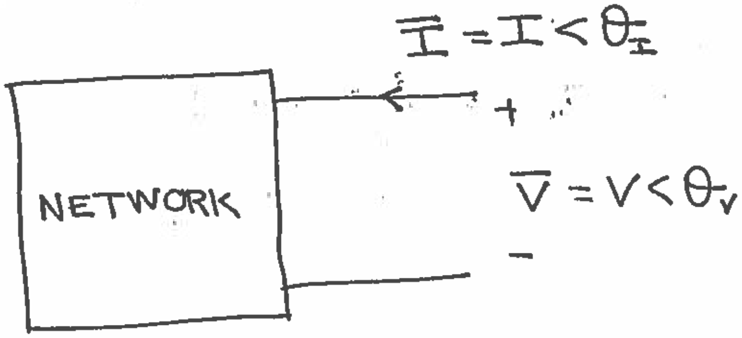
$$\left.\begin{aligned} \overline{S_1} &= \overline{V_1} \cdot \overline{I_1}^* \\ \overline{S_2} &= \overline{V_2} \cdot \overline{I_2}^* \\ &\vdots \\ \overline{S_n} &= \overline{V_n} \cdot \overline{I_n}^* \end{aligned}\right\} \longrightarrow \sum_{j=1}^{n} \overline{S_j} = \overline{V_1} \cdot \overline{I_1}^* + \overline{V_2} \cdot \overline{I_2}^* + \ldots + \overline{V_n}\,\overline{I_n}^*$$

$$= \overline{V} \left(\overline{I_1}^* + \overline{I_2}^* + \ldots + \overline{I_n}^* \right)$$

$$= \overline{V} \underbrace{\left(\overline{I_1} + \overline{I_2} + \ldots + \overline{I_n} \right)^*}_{0}$$

$\overline{V_1} = \overline{V_2} = \ldots = \overline{V_n} = \overline{V}$

REACTIVE POWER COMPENSATION

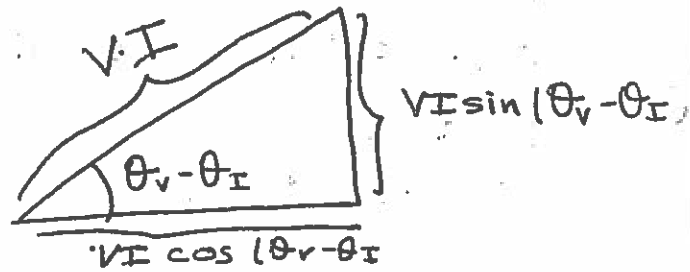


$$\bar{S} = \bar{V}\cdot\bar{I}^* = P + jQ$$

$$P = VI\cos(\theta_V - \theta_I)$$

$$Q = VI\sin(\theta_V - \theta_I)$$

$$S = \sqrt{P^2 + Q^2}$$

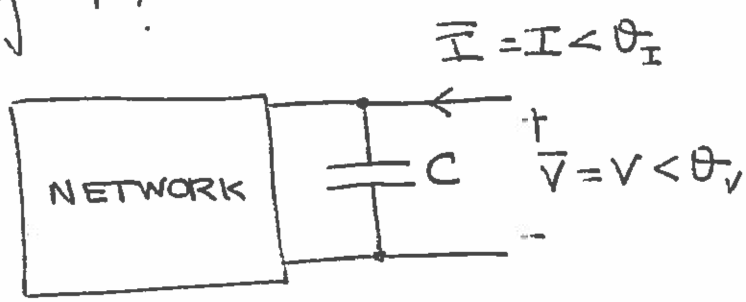


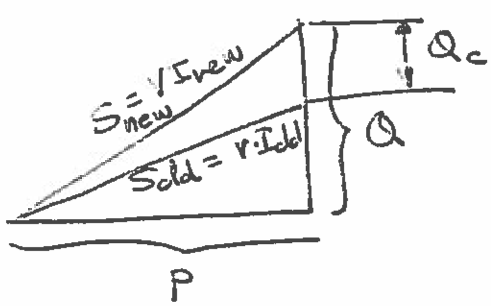
POWER TRIANGLE

- The nature of the network determines $\theta_V - \theta_I$ → P, Q

⇒ for a fixed V, we will have ~~a~~ a corresponding I

- Is there a way we can reduce $\bar{S}$ while maintaining P?





$$S_{new} = \sqrt{P^2 + (Q - Q_c)^2} = V I_{new}$$

$$Q_c = \frac{V^2}{X_c}$$

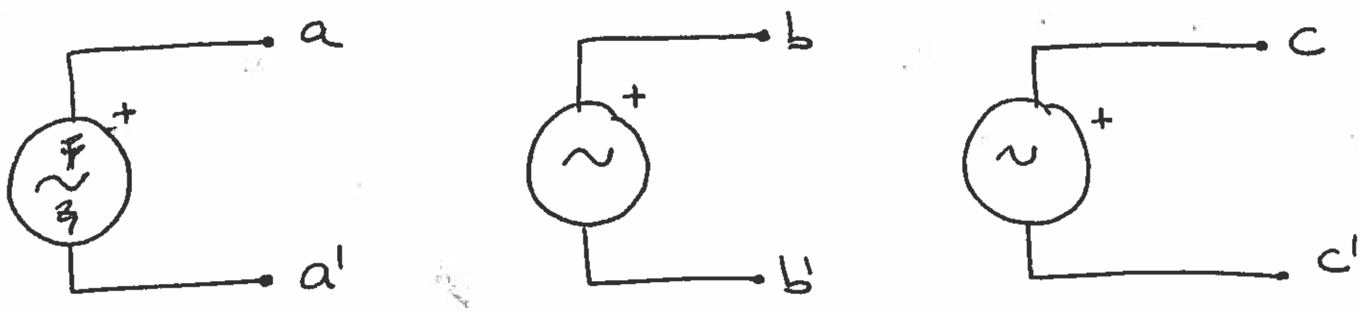
$S_{new} = V I_{new}$

$S_{old} = V I_{old}$

⇒ $S_{new} < S_{old}$

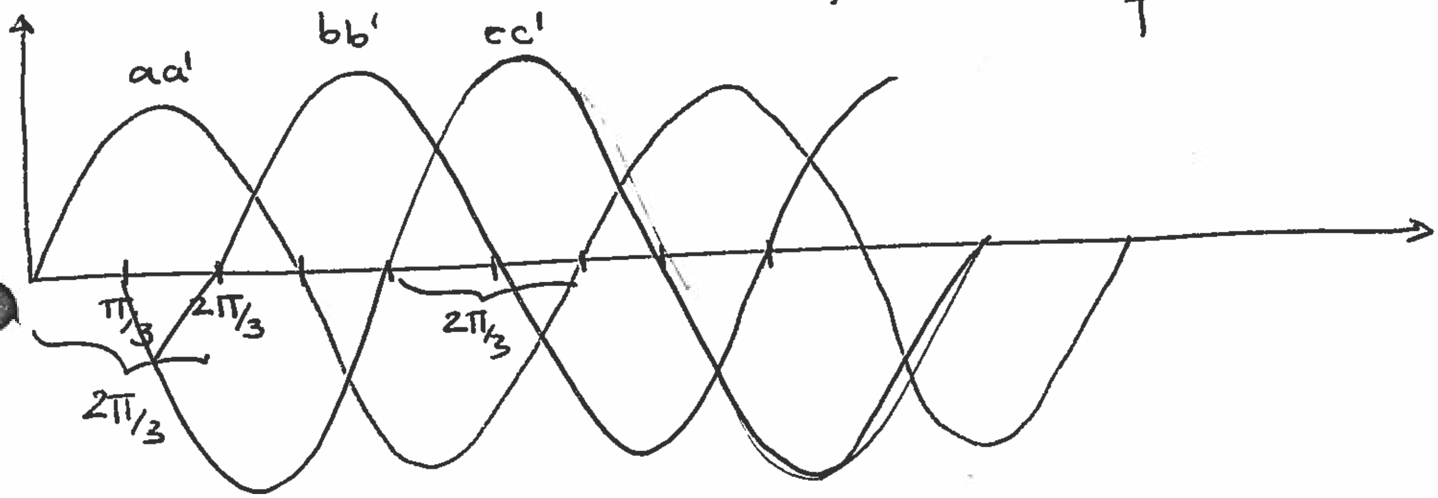
⇒ $I_{new} < I_{old}$.

THREE-PHASE SYSTEMS

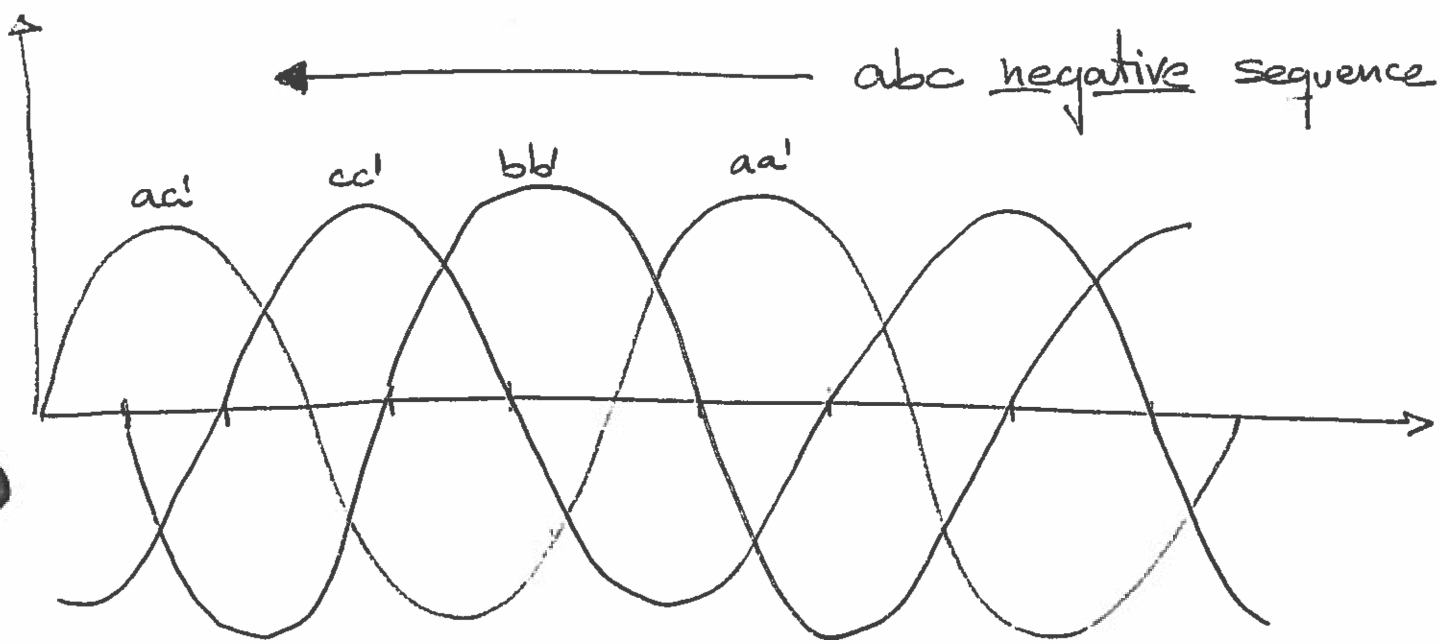


$V_{aa'}(t) = \sqrt{2}V\cos(\omega t)$ $\quad V_{bb'}(t) = \sqrt{2}V\cos(\omega t \mp \frac{2\pi}{3})$ $\quad V_{cc'}(t) = \sqrt{2}V\text{ca}\ \pm\frac{2\pi}{3}$

$\longrightarrow$ abc <u>positive</u> sequence

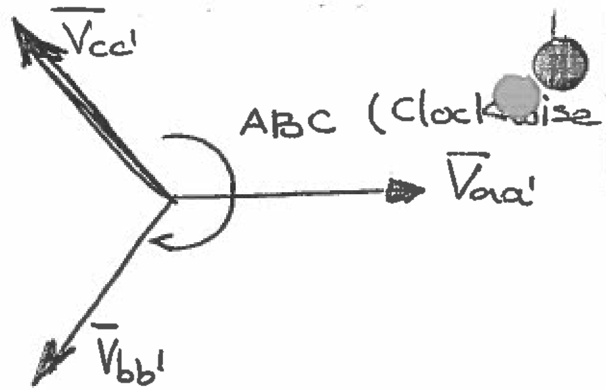


$\longleftarrow$ abc <u>negative</u> sequence

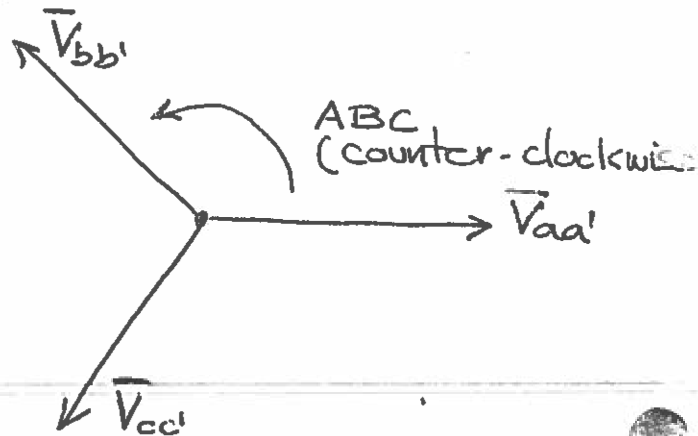


PHASOR REPRESENTATION

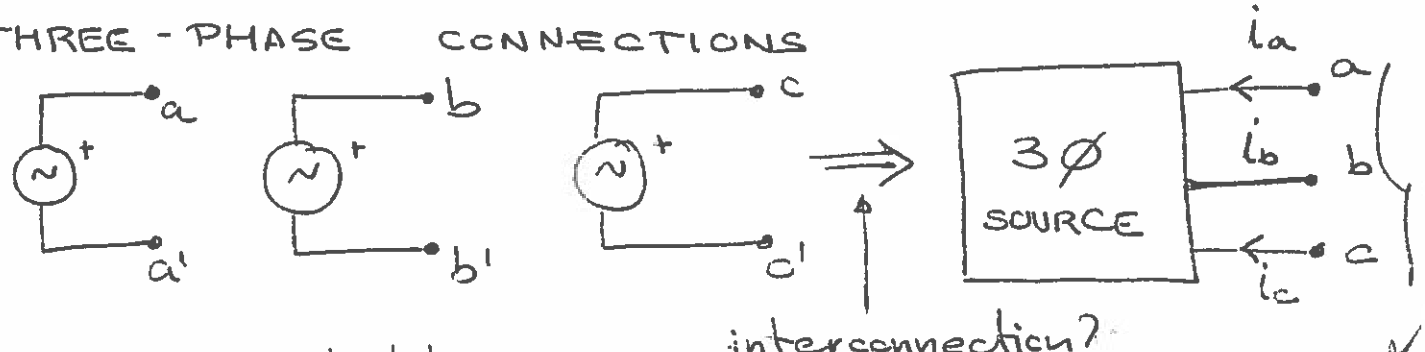
$$\oplus \begin{cases} \overline{V}_{aa'} = V \angle \theta_v = V \angle 0 \\ \overline{V}_{bb'} = V \angle -2\pi/3 \\ \overline{V}_{cc'} = V \angle \dfrac{2\pi}{3} \end{cases}$$



$$\ominus \begin{cases} \overline{V}_{aa'} = V \angle \theta_v \\ \overline{V}_{bb'} = V \angle \dfrac{2\pi}{3} \\ \overline{V}_{cc'} = V \angle -\dfrac{2\pi}{3} \end{cases}$$



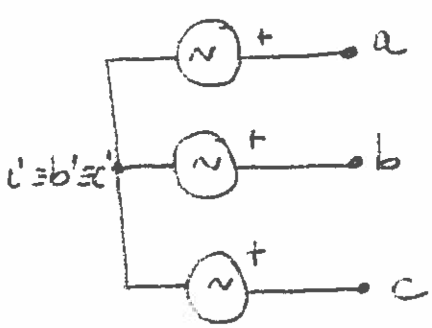
THREE-PHASE CONNECTIONS

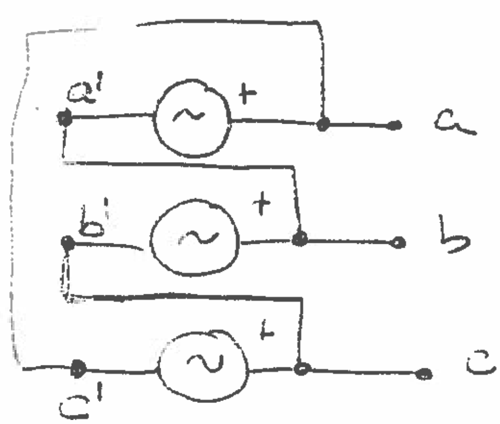


Three terminals
two ports.

Two possibilites

WYE CONNECTION





Definitions: rms values of:

i) Line voltage(V_L): voltage across any two term[inals]

ii) Phase voltage (V_ϕ): voltage across the termin[als] of a source

iii) Line current (I_L): current flowing out of a terminal

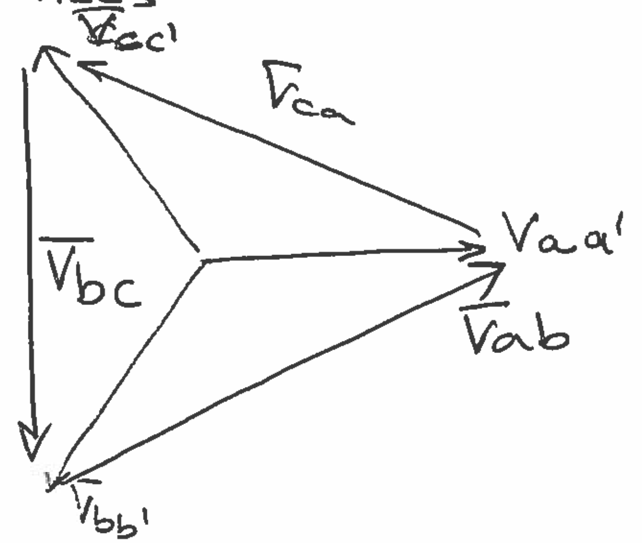
iv) Phase current. (I_ϕ): current flowing across a source

Y- connection (if balanced operation) $V_L = \sqrt{3} V_\phi$, $I_L = I_\phi$

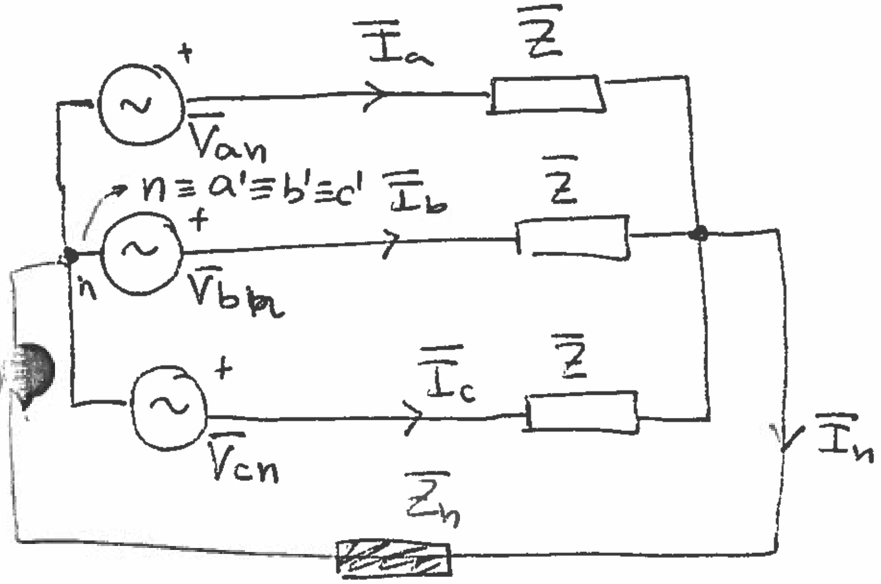
Δ- connection $V_L = V_\phi$, $I_L = \sqrt{3} I_\phi$

RELATIONS BETWEEN LINE & PHASE VOLTAGES FOR Y- CONNECTED SOURCES

$$\overline{V}_{ab} = \overline{V}_{aa'} - \overline{V}_{bb'}$$
$$\overline{V}_{bc} = \overline{V}_{bb'} - V_{cc'}$$
$$\overline{V}_{ca} = \overline{V}_{cc'} - V_{aa'}$$



Y-Y CONNECTION W/ BALANCED LOAD



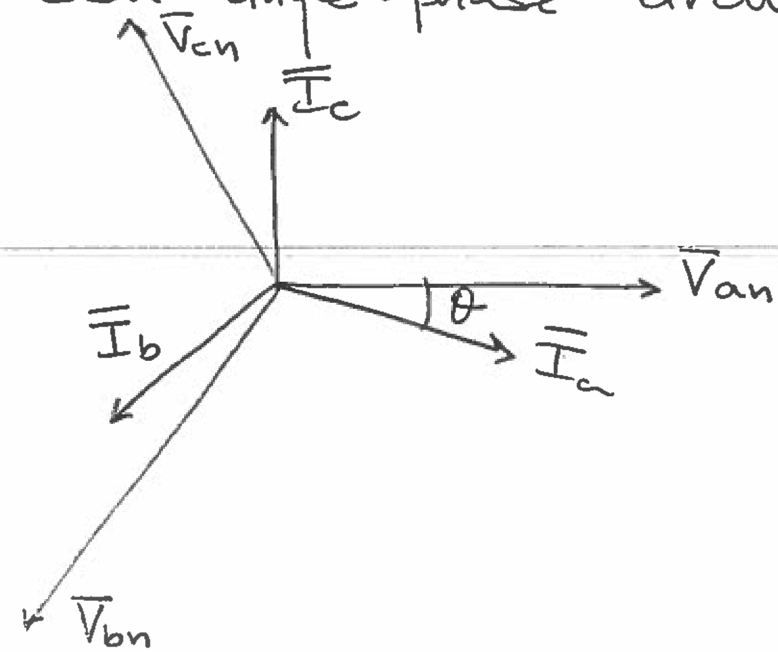
$$\overline{V}_{an} = \overline{Z}\,\overline{I}_a + \overline{Z}_n \overline{I}_n$$
$$\overline{V}_{bn} = \overline{Z}\,\overline{I}_b + \overline{Z}_n \overline{I}_n$$
$$\overline{V}_{cn} = \overline{Z} \cdot \overline{I}_c + \overline{Z}_n \overline{I}_n$$

$$\underbrace{\bar{V}_{an} + \bar{V}_{bn} + V_{cn}}_{0} = \bar{Z}(\underbrace{\bar{I}_a + \bar{I}_b + \bar{I}_c}_{\bar{I}_n}) + 3\bar{I}_n \cdot \bar{Z}_n$$

$$(\bar{Z} + 3\bar{Z}_n)\bar{I}_n = 0 .$$

$$\text{if } \bar{Z} + 3\bar{Z}_n \neq 0 \longrightarrow \bar{I}_n = 0$$

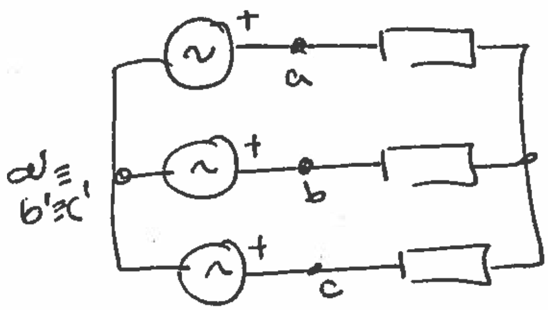
This analysis suggest that we can analyze each single-phase circuit independently.



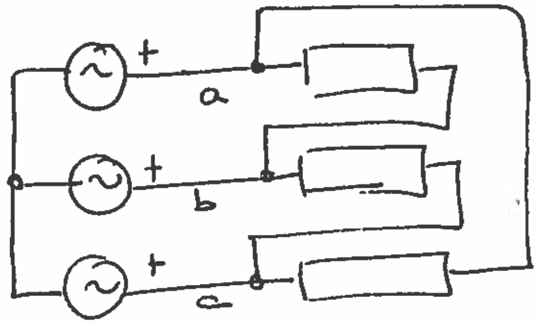
<u>OBSERVATION</u>

The currents also form a three-phase balance set.

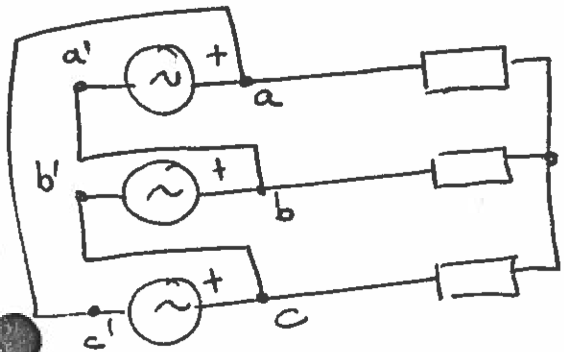
OTHER THREE PHASE CONNECTIONS



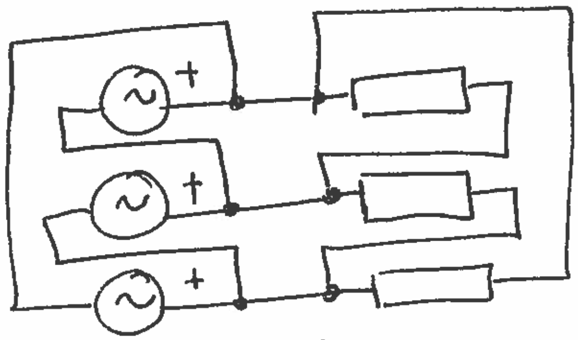
Y-Y



Y-Δ



Δ-Y



Δ-Δ

THREE-PHASE POWER

Y:

$$\overline{V}_{an} = \sqrt{2}\, V\cos(\omega t) \qquad \overline{I}_a = \sqrt{2}\, I\cos(\omega t - \theta)$$

$$\overline{V}_{bn} = \sqrt{2}\, V\cos\left(\omega t - \frac{2\pi}{3}\right) \qquad \overline{I}_b = \sqrt{2}\, I\cos\left(\omega t - \theta - \frac{2\pi}{3}\right)$$

$$\overline{V}_{cn} = \sqrt{2}\, V\cos\left(\omega t + \frac{2\pi}{3}\right) \qquad \overline{I}_c = \sqrt{2}\, I\cos\left(\omega t - \theta + \frac{2\pi}{3}\right)$$

$$P_a(t) = VI\left[\cos\theta + \cos(2\omega t - \theta)\right]$$

$$P_b(t) = VI\left[\cos\theta + \cos\left(2\omega t - \frac{4\pi}{3} - \theta\right)\right]$$

$$P_c(t) = VI\left[\cos\theta + \cos\left(2\omega t + \frac{4\pi}{3} - \theta\right)\right]$$

$$P(t) = 3VI\cos\theta + VI\underbrace{\left(\cos(2\omega t - \theta) + \cos\left(2\omega t - \frac{4\pi}{3} - \theta\right) + \cos\left(2\omega t + \frac{4\pi}{3} - \theta\right)\right)}_{0}$$

$$= 3VI\cos\theta$$

$$\boxed{P(t) = 3\,VI\cos\theta}$$ CONSTANT

$V = V_\phi$

$I = I_\phi$

IN Y– CONN : $V_\phi = \dfrac{V_L}{\sqrt{3}}$ $\quad I_\phi = I_L$

$$P = \sqrt{3}\,V_L I_L \cos\theta$$

Watch out

This is not the angle between ~~any~~ $\underbrace{\overline{V}_{ab}}_{V_L}$ and $\underbrace{\overline{I}_a}_{I_L}$!!

$$\overline{S} = 3VI\cos\theta + j\,VI\sin\theta$$

$$\overline{S} = \sqrt{3}\,V_L I_L\,(\cos\theta + j\sin\theta)$$

$$\boxed{S = \sqrt{3}\,V_L I_L}$$