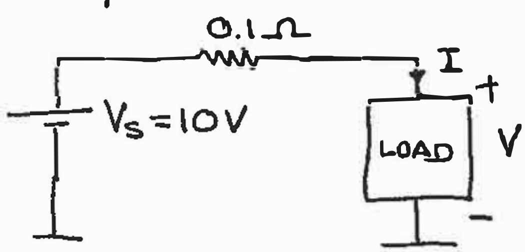
08/29/17

LECTURE 1

- READING: Chapters 1 & 2 of SG&O
- THE CENTRAL PROBLEM IN POWER SYSTEMS
- REVIEW OF PHASORS
- POWER: INSTANTENOUS, AVERAGE & COMPLEX

THE CENTRAL PROBLEM IN POWER SYSTEMS

- It is to compute all electric variables given load consumptions. In a nutshell with a simple DC system



LOAD $P = 10W$, what are the values of V & I?

Power:

$$\left.\begin{array}{l} V \cdot I = P = 10 \\ V_s = 0.1 \cdot I + V \\ \overset{\shortparallel}{10V} \end{array}\right\} \to \left.\begin{array}{l} I = \dfrac{10 - V}{0.1} \\ 10 = V \cdot I \end{array}\right\} \to 10 = V \cdot \left(\frac{10 - V}{0.1}\right) \to$$

$$\to 1 = 10V - V^2 = 0 \to V^2 - 10V + 1 = 0$$

$$V = \frac{10 \pm \sqrt{10^2 - 4}}{2} = \begin{cases} \dfrac{10 + \sqrt{96}}{2} = 9.8990V \to I = 1.01A \\[2ex] \dfrac{10 - \sqrt{96}}{2} = 0.1010V \to I = 98.99A \end{cases}$$

Both solutions are possible in real life.

- The first one represents a case when the system is working properly (good voltage on the load, small currents)

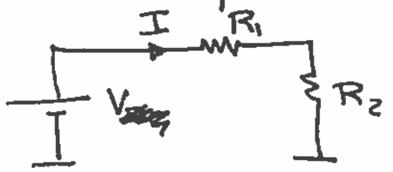
- The second case is typical of a situation where there could be a fault (a short-circuit). We are interested in understanding this case to size properly conductors and to set protections properly.

REVIEW OF PHASORS

- The systems we will study extensively in this class are AC electric power systems.
- The use of phasors is very convenient to ~~solve~~ formulate and solve problems of interest.

- Let's go back to the DC example.

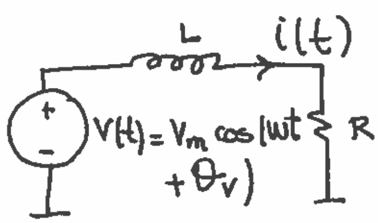
(I)



DC system

$V = (R_1 + R_2) I$ → this is an algebraic expression, which we know how to manipulate.

(II)



Kirchoff's laws always apply, i.e., the physics holds, so we can write the following diff eq:

$$V(t) = R \cdot i(t) + L \cdot \frac{di}{dt}$$

Q: Can we use same trick to represent (II) in such a way that we can use algebraic representation instead of differential equation? Hopefully, this will make our lives easier

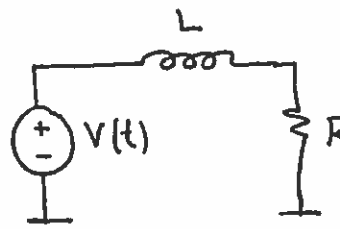
A: YES! PHASORS!!

$v(t) \longrightarrow \overline{V}$

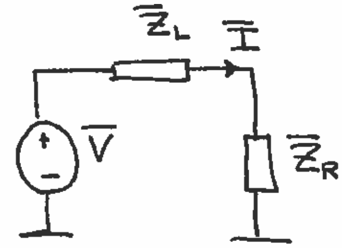
$L \longrightarrow \overline{Z}_L$

$R \longrightarrow \overline{Z}_R$

$i \longrightarrow \overline{I}$

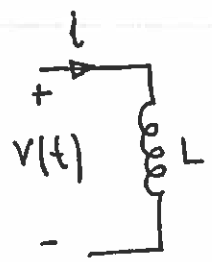


$\Rightarrow$



$$v(t) = L \cdot \frac{di}{dt} + R \cdot i \longrightarrow \overline{V} = (\overline{Z}_L + \overline{Z}_R)\overline{I}$$

Q: How do we go from $v(t), L, R$ to $\overline{V}, \overline{Z}_L, \overline{Z}_R$?



$$v(t) = L \cdot \frac{di}{dt}$$

$$v(t) = V_m \cos(\omega t + \theta_v) = L \cdot \frac{di}{dt}$$

In steady-state: $i(t) = I_m \cdot \cos(\omega t + \theta_i)$

$$V_m \cos(\omega t + \theta_v) = -L \cdot I_m \omega \sin(\omega t + \theta_i)$$

But $-\sin(\omega t + \theta_i) = \cos\left(\underset{+\omega t}{\theta_i} + \frac{\pi}{2}\right)$

$$V_m \cos(\omega t + \theta_v) = L \cdot I_m \omega \cdot \cos\left(\omega t + \theta_i + \frac{\pi}{2}\right)$$

$$\boxed{I_m = \frac{V_m}{\omega L} \qquad \theta_i = \theta_v - \frac{\pi}{2}}$$

Now let's add and substract $j V_m \sin(\omega t + \theta_v)$

and $j L I_m \omega \cos(\omega t + \theta_i + \frac{\pi}{2})$:

$$V_m \cdot \cos(\omega t + \theta_v) + j \sin(\omega t + \theta_v) = L \cdot I_m \cdot \omega \cos\left(\omega t + \theta_i + \frac{\pi}{2}\right) + $$
$$+ j \, \cancel{L} \, I_m \omega \sin(\omega t + \theta_i + \pi/2)$$

Euler's expansion:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Then:

$$V_m e^{j(\theta_v + \omega t)} = L\cdot\omega I_m e^{j(\theta_i + \omega t + \pi/2)}$$

$$V_m e^{j\theta_v}\cdot\cancel{e^{j\omega t}} = L\omega I_m e^{j(\theta_i + \pi/2)}\cancel{e^{j\omega t}} = L\cdot\omega I_m e^{j\pi/2}\cdot e^{j\theta_i} =$$

$$= L\omega\cdot e^{j\pi/2} I_m e^{j\theta_i}$$

Let's define: $\overline{V} \triangleq \dfrac{V_m}{\sqrt{2}} e^{j\theta_v}$

$$\overline{I}_m \triangleq \frac{I_m}{\sqrt{2}} e^{j\theta_i}$$

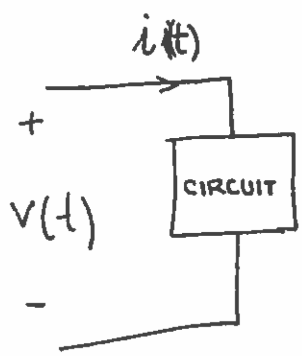
$$\overline{Z}_L \triangleq j\omega L$$

So we get: $\overline{V} = \overline{Z}_L \cdot \overline{I}$!

Similarly, we can get: $C \rightarrow \dfrac{1}{j\omega C}$

$$R \rightarrow R$$

INSTANTANEOUS POWER



$$P(t) = v(t)\cdot i(t)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$P(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

ROOT-MEAN SQUARE

- $$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \frac{V_m}{\sqrt{2}}$$

- Instead of $\bar{V} = \frac{V_m}{\sqrt{2}} < \theta_v$, we can define $\bar{V}$ as : $\bar{V} = V_{RMS} < \theta_v$.

- Average power then becomes:

$$P = V_{RMS} \cdot I_{RMS} \cdot \cos(\theta_v - \theta_i)$$

From now onwards, we drop RMS and just use V to represent the RMS value of $V(t) = V_m \cos(\omega t + \theta_v)$

COMPLEX POWER

Let's do a similar calculation, but instead of using $v(t)$ and $i(t)$, let's use $\bar{V}$ and $\bar{I}$

$$\bar{S} \triangleq \bar{V}\bar{I}^* = V e^{j\theta_v} \cdot I e^{-j\theta_i} = V \cdot I e^{j(\theta_v - \theta_i)}$$

$$\bar{S} = \overbrace{\underbrace{VI[\cos(\theta_v - \theta_i)}_{P} + j\, VI \sin(\theta_v - \theta_i)}^{Q}]$$

$\boxed{Q \triangleq VI \sin(\theta_v - \theta_i)}$ ← Reactive power

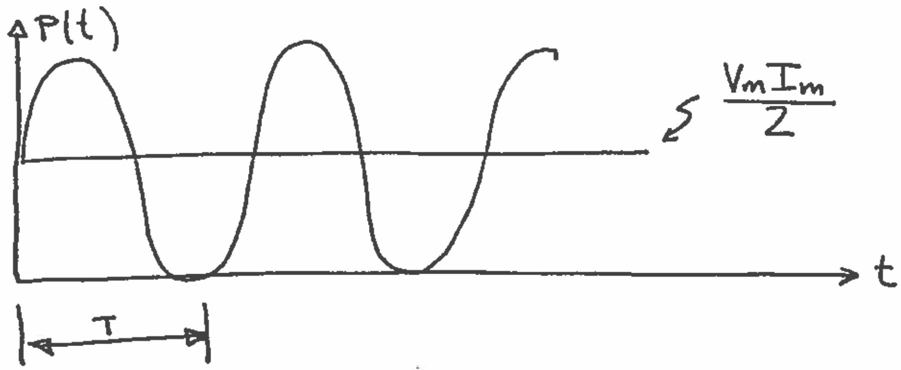
→ doesn't do anything for us but it is extremely important for maintaining voltage.

Reminder: $\cos(\theta_1)\cdot\cos\theta_2 = \frac{1}{2}\left[\cos(\theta_1-\theta_2)+\cos(\theta_1+\theta_2)\right]$

Thus:

$$P(t) = \frac{1}{2} V_m I_m \cdot \left[\cos(\theta_v-\theta_i) + \cos(2\omega t+\theta_v+\theta_i)\right]$$

$$P(t) = \frac{V_m I_m}{2}\cos(\theta_v-\theta_i) + \frac{V_m I_m}{2}\cos(2\omega t+\theta_v+\theta_i)$$



AVERAGE POWER

$$P = \frac{1}{T}\int_0^T \frac{VI}{2}\left[\cos(\theta_v-\theta_i)+\cos(2\omega t+\theta_v+\theta_i)\right]dt =$$

$$= \frac{V_m I_m}{2}\cos(\theta_v-\theta_i) + \frac{1}{T}\int_0^T \cos(2\omega t+\theta_v+\theta_i)\,dt =$$

$$= \frac{V_m I_m}{2}\cos(\theta_v-\theta_i) + \frac{1}{2\omega T}\cancelto{0}{\sin(2\omega t+\theta_v+\theta_i)\Big|_0^T}$$

$$\boxed{P = \frac{V_m I_m}{2}\cos(\theta_v-\theta_i)}$$

POWER FACTOR

$$\boxed{PF \triangleq \cos(\theta_v-\theta_i)}$$