ECE 476 – Power System Analysis Fall 2017
Homework 1

In-class quiz: Thursday, September 7, 2017
Reading: Chapters 1 and 2 of GS&O

Problem 1. With $|V| = 100$ V, the instantaneous power $p(t)$ into a network N has a maximum value 1707 W and a minimum value of -293 W.

1. Find a possible series RL circuit equivalent to N.

   Define $v(t)$ and $i(t)$ of the network N as

$$v(t) = \sqrt{2}V\cos(\omega t + \theta_V),$$
$$i(t) = \sqrt{2}I\cos(\omega t + \theta_I).$$

   Then, the instantaneous power into N is

$$\begin{aligned} p(t) = v(t)i(t) &= 2VI\cos(\omega t + \theta_V)\cos(\omega t + \theta_I) \\ &= VI\left[\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)\right]. \end{aligned}$$

   The maximum value of $p(t)$ occurs when $\cos(2\omega t + \theta_V + \theta_I) = 1$ and, similarly, the minimum value of $p(t)$ occurs when $\cos(2\omega t + \theta_V + \theta_I) = -1$. So we have the following 2 equations:

$$P_{\text{max}} = VI\cos(\theta_V - \theta_I) + VI = 1707 \text{ W}$$
$$P_{\text{min}} = VI\cos(\theta_V - \theta_I) - VI = -293 \text{ W}$$

   Subtracting one from the other, we get

$$2VI = 1707 + 293 \implies VI = 1000 \text{ W} \implies I = 10 \text{ A},$$

   and adding the two and substituting $VI = 1000$ in, we get

$$2VI\cos(\theta_V - \theta_I) = 2000\cos(\theta_V - \theta_I) = 1414 \implies \theta_V - \theta_I = \cos^{-1}\left(\frac{1414}{2000}\right) = \pm 45°.$$

   An inductive load causes current to lag the voltage, so we get $\theta_V - \theta_I = 45°$. Now we use $V = 100\angle\theta_V$ and $I = 10\angle\theta_I$ to get

$$Z = \frac{V}{I} = \frac{100\angle\theta_V}{10\angle\theta_I} = 10\angle(\theta_V - \theta_I) = 10\angle 45° \ \Omega.$$

   Finally we obtain $R = \mathbb{R}(Z) = 7.07\ \Omega$ and $\omega L = \mathbb{I}(Z) = 7.07\ \Omega \implies L = 0.0188$ H.

2. Find $S = P + jQ$ into N.

   Using the voltage and current phasors from above, we get

$$S = VI^* = (100\angle\theta_V)(10\angle\theta_I)^* = (100\angle\theta_V)(10\angle(-\theta_I)) = 1000\angle(\theta_V - \theta_I) = 1000\angle 45°.$$

   So $S = P + jQ$, where $P = \mathbb{R}(S) = 707$ W and $Q = \mathbb{I}(S) = 707$ Var.

3. Find the maximum instantaneous power into L and compare with Q.

   The instantaneous power into L is

$$\begin{aligned} p_L(t) = v_L(t)i(t) = L\frac{di}{dt}i(t) &= -L\sqrt{2}\omega I\sin(\omega t + \theta_I)\sqrt{2}I\cos(\omega t + \theta_I) \\ &= 2\omega LI^2\sin(\omega t + \theta_I)\cos(\omega t + \theta_I) \\ &= \omega LI^2\sin(2\omega t + 2\theta_I). \end{aligned}$$

   The maximum value of $p_L(t)$ occurs when $\sin(2\omega t + 2\theta_I) = 1$, and at this point,

$$p_{L,\max} = \omega LI^2 = 2\pi 60(0.0188)(10^2) = 707 \text{ W},$$

   which is equal to Q.

Problem 2. A certain 1ϕ load draws 5 MW at 0.7 power factor lagging. Determine the reactive power required from a parallel capacitor to bring the power factor of the parallel combination up to 0.9.

With the current power factor of 0.7 lagging, we solve the following for the current Q:

$$\tan(\cos^{-1}(0.7)) = \frac{Q_{cur}}{P} = \frac{Q_{cur}}{5} \implies Q_{cur} = 5.101 \text{ MVar}$$

To reach a power factor of 0.9 lagging, we solve the following for the desired Q:

$$\tan(\cos^{-1}(0.9)) = \frac{Q_{des}}{P} = \frac{Q_{des}}{5} \implies Q_{des} = 2.422 \text{ MVar}$$

Therefore, the reactive power required from a parallel capacitor to bring the power factor to 0.9 is

$$Q_{cap} = Q_{cur} - Q_{des} = 5.101 - 2.422 = 2.679 \text{ MVar}$$

Problem 3. A 3ϕ load draws 200 kW at a PF of 0.707 lagging from a 440-V line. In parallel is a 3ϕ capacitor bank that supplies 50 kVAr. Find the resultant power factor and current (magnitude) into the parallel combination.

In each phase, the load draws 200/3 kW at a PF of 0.707 lagging. So we solve for the reactive power that the load draws in each phase as follows:

$$\tan(\cos^{-1}(0.707)) = \frac{Q_{load,1\phi}}{P_{load,1\phi}} = \frac{Q_{load,1\phi}}{200/3} \implies Q_{load,1\phi} = 66.69 \text{ kVar}.$$

With the capacitor bank in parallel, the combined reactive power drawn becomes

$$Q_{combo,1\phi} = Q_{load,1\phi} - Q_{cap,1\phi} = 66.69 - 50/3 = 50.02 \text{ kVar}.$$

So the power factor of the combination is

$$\cos\left(\tan^{-1}\left(\frac{50.02}{200/3}\right)\right) = 0.7999 \approx 0.8 \text{ lagging}$$

The current magnitude into the combination is

$$|I_{combo,1\phi}| = \frac{|S_{combo,1\phi}|}{|V|} = \frac{\sqrt{P_{1\phi}^2 + Q_{combo,1\phi}^2}}{|V|} = \frac{\sqrt{(200/3)^2 + 50.02^2}}{440} = 189 \text{ A, per phase}$$

Problem 4. A 1ϕ load draws 10 kW from a 416-V line at a power factor of 0.9 lagging.

1. Find $S = P + jQ$.

   At power factor 0.9 lagging, the complex power drawn is solved as

   $$\tan(\cos^{-1}(0.9)) = \frac{Q}{P} = \frac{Q}{10} \implies Q = 4.84 \text{ kVar}.$$

   Then, $S = 10 + j4.84$ kVA.

2. Find $|I|$.

   $$|I| = \frac{|S|}{|V|} = \frac{\sqrt{10^2 + 4.84^2} \times 1000}{416} = 26.7 \text{ A}$$

3. Assume that $\angle I = 0$ and find the instantaneous power $p(t)$.

   $$\begin{aligned} p(t) = v(t)i(t) &= \sqrt{2}V\cos(\omega t + \theta_V)\sqrt{2}I\cos(\omega t) \\ &= 2VI\cos(\omega t + \theta_V)\cos(\omega t) \\ &= VI\cos\theta_V + VI\cos(2\omega t + \theta_V) \\ &= P + VI\left[\cos(2\omega t)\cos\theta_V - \sin(2\omega t)\sin\theta_V\right] \\ &= P + VI\cos\theta_V\cos(2\omega t) - VI\sin\theta_V\sin(2\omega t) \\ &= P + P\cos(2\omega t) - Q\sin(2\omega t) \\ &= P(1+\cos(2\omega t)) - Q\sin(2\omega t) \\ &= 10(1+\cos(2\omega t)) - 4.84\sin(2\omega t) \text{ kW} \end{aligned}$$

Problem 5. A small manufacturing plant is located 2km down a transmission line, which has a series reactance of 0.5 Ω/km. The line resistance is negligible. The line voltage plant is $480\angle 0$ V (rms), and the plant consumes 120 kW at 0.85 power factor lagging. Determine the voltage and power factor at the sending end of the transmission line by using:

1. A complex power approach.

   The load draws 120 kW at 0.85 power factor lagging. We solve for the reactive power drawn by the load as

   $$\tan(\cos^{-1}(0.85)) = \frac{Q_{load}}{P_{load}} = \frac{Q_{load}}{120} \implies Q_{load} = 74.37 \text{ kVar}.$$

   Therefore, the complex power drawn by the load is $S_{load} = 120 + j74.37$ kVA. We can now solve for the current into the load as

   $$I = \left(\frac{S_{load}}{V_{load}}\right)^* = \left(\frac{120 + j74.37}{480\angle 0^\circ}\right)^* = 294.1\angle(-31.79^\circ) \text{ A}.$$

The loss in the line can be computed as

$$S_{line} = V_{line} I_{line}^{*} = Z_{line} I_{load} I^{*} = j2(0.5)294.1^2 = j86.51 \text{ kVA}.$$

Thus, the complex power supplied by the source is

$$S_{source} = S_{load} + S_{line} = 120 + j74.37 + j86.51 = 120 + j160.88 \text{ kVA} = 200.7\angle 53.28^\circ \text{ kVA}.$$

So the power factor at the sending end is $\cos(53.28^\circ) = 0.598$, lagging.

Finally, the voltage at the sending end is

$$V_{source} = \frac{S_{source}}{I^{*}} = \frac{200.7\angle 53.28^\circ}{294.1\angle 31.79^\circ} = 682.4\angle 21.5^\circ \text{ V}.$$

2. A circuit analysis approach.

   Using KVL, we have

$$V_{source} = Z_{line} I + V_{load} = j1(294.1\angle(-31.79^\circ)) + 480 = 682.4\angle 21.5^\circ \text{ V}.$$

   And the power factor is $\cos(\theta_V - \theta_I) = \cos(21.5^\circ + -31.79^\circ) = 0.598$, lagging.