Maintaining visibility for leader-follower formations in obstacle environments

Dimitra Panagou and Vijay Kumar

***Abstract*— This paper addresses the problem of controlling a leader-follower $(\mathrm{L}-\mathrm{F})$ formation of two unicycle mobile robots moving under visibility constraints in a known obstacle environment. Visibility constraints are realized as inequality state constraints that determine a visibility set K. Maintaining visibility is translated into controlling the robots so that system trajectories starting in K always remain in K. We provide the conditions under which visibility is maintained, as well as a feedback control scheme that forces F to converge and remain into a set of desired configurations w.r.t. L while maintaining visibility. We also propose a cooperative control scheme for the motion of the formation in a known obstacle environment, so that both collision avoidance and maintaining visibility are ensured. The proposed control schemes are decentralized, in the sense that there is no direct communication between the robots. The efficacy of our algorithms is evaluated through simulations.**

I. Introduction

Control of leader-follower $(\mathrm{L}-\mathrm{F})$ formations has seen an increasing interest during the past few years, stimulated in part by the recent technological advances in communications and computation, which have allowed for the development of multi-agent systems that accomplish tasks effectively and reliably. Within the field of mobile robotics in particular, $\mathrm{L}-\mathrm{F}$ formations arise in applications ranging from surveillance and inspection to exploration and coverage. From a practical point of view, the case when limited sensing and/or limited communication among the robots are imposed is of particular interest; for instance, a very likely scenario for mobile robots operating in indoor environments is that global state feedback is not available to all robots, or that communication among them is restricted. These specifications impose various types of constraints to each robot, extending to the whole system, and should be taken into account during the control design.

This paper considers the case of two mobile robots with unicycle kinematics that operate in a known environment with obstacles while communication between them is not available. Assume that one of the robots (the leader L) is given a high-level motion plan for moving from an initial to a goal state in the free space. The task for the second robot (the follower F) is to move while keeping a fixed distance and orientation with respect to (w.r.t.) L, using the information from an onboard camera only, while also avoiding collisions.

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The problem of maintaining formations using vision-based control is quite popular [1]–[3]; in these contributions the robots are assumed to have omnidirectional cameras and that the velocities of L are either communicated to, or estimated by F. However, when communication is absent and the onboard sensors have limited capabilities (e.g. limited range and angle-of-view), the robots can stay connected if and only if L is always in the field-of-view (f.o.v.) of F. The latter specification imposes a set of visibility constraints, which should never be violated so that F maintains visibility with L. The problem of controlling a nonholonomic robot so that it maintains visibility with a fixed or moving target has been of increasing interest, see [4]–[7]. Moreover, when the robots operate in obstacle environments, avoiding collisions with obstacles, as well as between robots should be guaranteed.

This paper proposes a feedback control solution for a $\mathrm{L}-\mathrm{F}$ formation of two unicycle mobile robots, that move in a known obstacle environment under visibility constraints and without explicit communication between them. Visibility constraints are realized as nonlinear inequalities in terms of the system states, that determine a closed subset K of the state space called the visibility set K. Maintaining visibility can thus be translated into controlling the robots so that system trajectories starting in K always remain in K (Section II). Inspired by ideas from viability theory [8], we provide the necessary conditions for visibility maintenance, as well as a control scheme that forces F to converge and remain into a set of desired configurations w.r.t. L while maintaining visibility (Section III). We also propose a cooperative control scheme for the $\mathrm{L}-\mathrm{F}$ motion in a known obstacle environment, so that both collision avoidance and maintaining visibility are ensured (Section IV). The proposed control schemes are decentralized, in the sense that there is no direct communication between the robots; the follower is localized w.r.t. L, however is aware neither of the leader's navigation plan, nor of its velocities at each time instant, while the leader is not aware of the follower's state. The efficacy of our algorithms is evaluated via simulation results.

II. Mathematical Modeling

A. Leader-Follower Kinematics

Consider two unicycle mobile robots moving in $\mathrm{L}-\mathrm{F}$ formation. The motion of each one of the robots L, F, w.r.t. a global frame $\mathcal{G}$ is described by

$$\dot{\boldsymbol{q}}_{\boldsymbol{i}} = \boldsymbol{G}(\boldsymbol{q}_{\boldsymbol{i}})\boldsymbol{v}_{\boldsymbol{i}} \Rightarrow \begin{bmatrix}\dot{x}_i\\ \dot{y}_i\\ \dot{\theta}_i\end{bmatrix} = \begin{bmatrix}\cos\theta_i & 0\\ \sin\theta_i & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_i\\ w_i\end{bmatrix}, \tag{1}$$

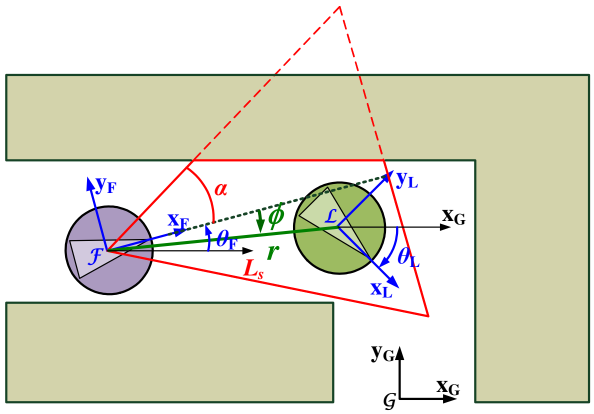


Fig. 1. The system setup in an obstacle environment

where $i \in \{\mathrm{L}, \mathrm{F}\}$, $\boldsymbol{q_i} = \begin{bmatrix} x_i & y_i & \theta_i \end{bmatrix}^{\mathrm{T}}$ is the configuration vector of i, $\boldsymbol{r_i} = \begin{bmatrix} x_i & y_i \end{bmatrix}^{\mathrm{T}}$ is the position vector and θ_i is the orientation of i w.r.t. frame $\mathcal{G}$, u_i, w_i are the linear and angular velocity of i in the body-fixed frame ($\mathcal{L}$ or $\mathcal{F}$).

Following [7], we describe the motion of F w.r.t. the leader frame $\mathcal{L}$; consider the position vector $\boldsymbol{r} = \begin{bmatrix} x & y \end{bmatrix}^{\mathrm{T}}$ of F w.r.t. $\mathcal{L}$, $\boldsymbol{r} = \mathbf{R}(-\theta_{\mathrm{L}})\,(\boldsymbol{r_{\mathrm{F}}} - \boldsymbol{r_{\mathrm{L}}})$, and take the time derivative

$$\dot{\boldsymbol{r}} = \dot{\mathbf{R}}(-\theta_{\mathrm{L}})\,(\boldsymbol{r_{\mathrm{F}}} - \boldsymbol{r_{\mathrm{L}}}) + \mathbf{R}(-\theta_{\mathrm{L}})\,(\dot{\boldsymbol{r}}_{\mathbf{F}} - \dot{\boldsymbol{r}}_{\mathbf{L}})\,, \tag{2}$$

where

$$\mathbf{R}(-\theta_{\mathrm{L}}) = \begin{bmatrix} \cos(-\theta_{\mathrm{L}}) & -\sin(-\theta_{\mathrm{L}}) \\ \sin(-\theta_{\mathrm{L}}) & \cos(-\theta_{\mathrm{L}}) \end{bmatrix} = \begin{bmatrix} \cos\theta_{\mathrm{L}} & \sin\theta_{\mathrm{L}} \\ -\sin\theta_{\mathrm{L}} & \cos\theta_{\mathrm{L}} \end{bmatrix} \tag{3}$$

is the rotation matrix of the frame $\mathcal{L}$ w.r.t. frame $\mathcal{G}$, and

$$\dot{\mathbf{R}}(-\theta_{\mathrm{L}}) = \begin{bmatrix} 0 & w_{\mathrm{L}} \\ -w_{\mathrm{L}} & 0 \end{bmatrix} \mathbf{R}(-\theta_{\mathrm{L}}). \tag{4}$$

Substituting (3), (4), (1) into (2) and after some algebra one eventually gets

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -1 & y \\ 0 & -x \end{bmatrix} \begin{bmatrix} u_{\mathrm{L}} \\ w_{\mathrm{L}} \end{bmatrix} + \begin{bmatrix} \cos(\theta_{\mathrm{F}} - \theta_{\mathrm{L}}) \\ \sin(\theta_{\mathrm{F}} - \theta_{\mathrm{L}}) \end{bmatrix} u_{\mathrm{F}}. \tag{5}$$

Define $\beta = \theta_{\mathrm{F}} - \theta_{\mathrm{L}}$; then differentiating w.r.t. time yields

$$\dot{\beta} = \dot{\theta}_{\mathrm{F}} - \dot{\theta}_{\mathrm{L}} = w_{\mathrm{F}} - w_{\mathrm{L}}. \tag{6}$$

Combining (5) and (6) yields the system of equations

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\beta} \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\beta & 0 \\ \sin\beta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{\mathrm{F}} \\ w_{\mathrm{F}} \end{bmatrix}}_{\boldsymbol{f}(\boldsymbol{q}, \boldsymbol{v_{\mathrm{F}}})} + \underbrace{\begin{bmatrix} -1 & y \\ 0 & -x \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_{\mathrm{L}} \\ w_{\mathrm{L}} \end{bmatrix}}_{\boldsymbol{g}(\boldsymbol{q}, \boldsymbol{v_{\mathrm{L}}})}, \tag{7}$$

where $\boldsymbol{q} = \begin{bmatrix} x & y & \beta \end{bmatrix}^{\mathrm{T}} \in \mathcal{C}$ is the state vector, including the position $\boldsymbol{r} = \begin{bmatrix} x & y \end{bmatrix}^{\mathrm{T}}$ and orientation β of F w.r.t. the leader frame $\mathcal{L}$, $\mathcal{C}$ is the state space, $\boldsymbol{v_{\mathrm{F}}} = \begin{bmatrix} u_{\mathrm{F}} & w_{\mathrm{F}} \end{bmatrix}^{\mathrm{T}} \in \mathcal{U}_{\mathrm{F}}$ is the vector of control inputs and $\boldsymbol{g}(\boldsymbol{q}, \boldsymbol{v_{\mathrm{L}}}) \in \mathbb{R}^3$ can be seen as a perturbation vector field, where $\boldsymbol{v_{\mathrm{L}}} = \begin{bmatrix} u_{\mathrm{L}} & w_{\mathrm{L}} \end{bmatrix}^{\mathrm{T}} \in \mathcal{U}_{\mathrm{L}}$ is the vector of control inputs of L. Note that the perturbation is vanishing if and only if $\boldsymbol{g}(\boldsymbol{q}, \boldsymbol{v_{\mathrm{L}}}) = \mathbf{0}$, which occurs if and only if $\boldsymbol{v_{\mathrm{L}}} = \mathbf{0}$. Consequently, the motion of L can be thought as a non-vanishing perturbation to F.

B. Visibility constraints

F is assumed to have an onboard camera with angle-of-view $2\alpha < \pi$, and that it can reliably detect objects which are within a maximum range L_s as shown in Fig. 1. These specifications define a "cone-of-view" for F, which essentially is an isosceles triangle (in obstacle-free environments). Assume also that F is localized w.r.t. L, i.e. that the distance $\|\boldsymbol{r}\| = \sqrt{x^2 + y^2}$, as well as the bearing angle $\phi \in (-\pi, \pi]$ are measured. Consequently, at each time instant t, F can detect L if and only L is in the cone-of-view, i.e.

$$|\phi| \le \alpha \quad \text{and} \quad r \le L_s(\phi) = \frac{L_s \cos\alpha}{\cos\phi}. \tag{8}$$

These constraints define a closed subset K of $\mathcal{C}$, given as

$$K = \{\boldsymbol{q} \in \mathcal{C} \mid h_k(\boldsymbol{q}) \le 0,\ k = 1, 2\}\,, \tag{9}$$

where $h_1 = |\phi| - \alpha$ and $h_2 = r - L_s(\phi)$, which we call the *visibility set* K. The set K includes every configuration $\boldsymbol{q}$ for which visibility is maintained. Then, controlling F, L so that the resulting trajectories $\boldsymbol{q}(t)$ never escape K, implies that visibility is always maintained.

Consequently, the problem of maintaining visibility reduces into finding control inputs $\boldsymbol{v_{\mathrm{F}}} \in \mathcal{U}_{\mathrm{F}}$ for F, such that the visibility constraints (8) are met $\forall t \ge 0$, despite the (non-vanishing) perturbation $\boldsymbol{g}(\boldsymbol{q}, \boldsymbol{v_{\mathrm{L}}})$ that is induced by L.

III. Control Design for the Perturbed System

Consider the perturbed system (7), where L is moving with $u_{\mathrm{L}} \neq 0$, $w_{\mathrm{L}} \neq 0$ in an obstacle-free environment. F is localized w.r.t. the L, i.e. the position (x, y) and orientation β w.r.t. the leader frame $\mathcal{L}$ is available to F; however F is neither aware of the leader's navigation plan, nor of the velocities $u_{\mathrm{L}}(t)$, $w_{\mathrm{L}}(t)$ at each time instant t. Therefore, it is reasonable to assume that F has some "a priori" knowledge on the velocity bounds of L, in the sense that L is restricted to move at most with velocities $|u_{\mathrm{L}}| \le u_{\mathrm{L\,M}}$, $|w_{\mathrm{L}}| \le w_{\mathrm{L\,M}}$.

The task for F is to keep a fixed distance r_d w.r.t. L with angle $\phi = 0$, where $2r_0 \le r_d \le L_s \cos\alpha$, and r_0 is the radius of the robots; in that way, L is centered in the camera f.o.v.. This requirement specifies a manifold $\mathcal{M}$ of desired configurations: $\boldsymbol{q_d} = \begin{bmatrix} x_d & y_d & \theta_d \end{bmatrix}^{\mathrm{T}}$ for F,

$$\mathcal{M} = \left\{ \boldsymbol{q_d} \in \mathcal{C} \;\middle|\; \begin{array}{c} {x_d}^2 + {y_d}^2 = {r_d}^2, \\ \theta_d = \mathrm{atan2}(y_d, x_d) + \mathrm{sign}(y_d)\pi \end{array} \right\}.$$

Thus, the control design for F reduces into finding a feedback control law so that F converges to a $\boldsymbol{q_d} \in \mathcal{M}$, while the trajectories $\boldsymbol{q}(t)$ satisfy the visibility constraints (8) $\forall t \ge 0$. However, the perturbation $\boldsymbol{g}(\boldsymbol{q}, \boldsymbol{v_{\mathrm{L}}})$ is non-vanishing $\forall \boldsymbol{q} \in \mathcal{C}$, and thus $\boldsymbol{q_d} \in \mathcal{M}$ is not an equilibrium point of (7). In that case, the best one can hope for is that the system trajectories $\boldsymbol{q}(t)$ are ultimately bounded [9].

Therefore, the task for F reads as to converge and remain into a ball $\mathcal{B}(\boldsymbol{r_d}, \epsilon_r)$ of radius $\epsilon_r > 0$ around a desired position $\boldsymbol{r_d} \in \mathcal{M}$. The control design is based on the concept of *dipolar* vector fields [10]. A dipolar vector field $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$ has integral lines that all lead to the origin

$(x, y) = (0, 0)$ of the global frame $\mathcal{G}$, is non-vanishing everywhere in $\mathbb{R}^2$ except for the origin, and is given as

$$\mathbf{F}(\boldsymbol{r}) = \lambda(\boldsymbol{p}^{\mathrm{T}}\boldsymbol{r})\boldsymbol{r} - \boldsymbol{p}(\boldsymbol{r}^{\mathrm{T}}\boldsymbol{r}), \tag{10}$$

where $\lambda \geq 2$, $\boldsymbol{p} \in \mathbb{R}^2$ and $\boldsymbol{r} = \begin{bmatrix} x & y \end{bmatrix}^{\mathrm{T}}$ is the position vector w.r.t. $\mathcal{G}$. The main characteristic of a dipolar vector field (10) is that its integral lines converge to $(0, 0)$ with the direction φ_p of the vector $\boldsymbol{p}$. Then, choosing the vector $\boldsymbol{p} = \begin{bmatrix} p_x & p_y \end{bmatrix}^{\mathrm{T}}$ such that $\varphi_p = \operatorname{atan2}(p_y, p_x) \triangleq \theta_d$, reduces the orientation control of the unicycle into forcing it to align with the integral curves of the dipolar vector field. Therefore, if $\boldsymbol{p}$ is assigned on a desired position $\boldsymbol{r_d} = \begin{bmatrix} x_d & y_d \end{bmatrix}^{\mathrm{T}} \in \mathcal{M}$, then one gets a dipolar vector field whose integral lines converge to $\boldsymbol{r_d}$ having the desired orientation $\varphi_p \triangleq \theta_d$ (Fig. 2). The analytic form of (10), where $\boldsymbol{r}$ is substituted by $\boldsymbol{r_1} = \begin{bmatrix} x_1 & y_1 \end{bmatrix}^{\mathrm{T}}$, $\boldsymbol{r_1} = \boldsymbol{r} - \boldsymbol{r_d}$ and $\lambda = 3$ reads:

$$\mathrm{F}_x = 2p_x {x_1}^2 - p_x {y_1}^2 + 3p_y x_1 y_1, \tag{11a}$$

$$\mathrm{F}_y = 2p_y {y_1}^2 - p_y {x_1}^2 + 3p_x x_1 y_1. \tag{11b}$$

A. Ultimate boundedness

Theorem 1: The trajectories $\boldsymbol{r}(t) = \begin{bmatrix} x(t) & y(t) \end{bmatrix}^{\mathrm{T}}$ of the perturbed system (7) enter and remain into a ball $\mathcal{B}(\boldsymbol{r_d}, \epsilon_r)$ around the desired position $\boldsymbol{r_d}$, under the control law $\boldsymbol{v_\mathrm{F}} = \begin{bmatrix} u_\mathrm{F} & w_\mathrm{F} \end{bmatrix}^{\mathrm{T}}$ where

$$u_\mathrm{F} = -k_1 \operatorname{sgn}\!\left(\boldsymbol{r_1}^{\mathrm{T}} \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix}\right) \|\boldsymbol{r_1}\| - \operatorname{sgn}(\boldsymbol{p}^{\mathrm{T}}\boldsymbol{r_1}) u_{\mathrm{L\,M}}, \tag{12a}$$

$$w_\mathrm{F} = -k_2(\beta - \varphi) + \dot{\varphi}, \tag{12b}$$

where $k_1, k_2 > 0$, $\varphi = \operatorname{atan2}(\mathrm{F}_y, \mathrm{F}_x)$ is the orientation of the vector field at (x, y), $u_{\mathrm{L\,M}}$ is the upper bound of the linear velocity of L and $\epsilon_r > \frac{|w_\mathrm{L}|}{\sqrt{k_1 k_2}}$, see the proof in [11]. A conservative, yet safe, ϵ_r can be taken for the bound $w_{\mathrm{L\,M}}$ of the angular velocity of L. Note also that if $w_\mathrm{L} \to 0$, then $\epsilon_r \to 0$ as well.

B. Maintaining Visibility

The control law (12) forces F to converge into a ball around a desired position $\boldsymbol{r_d} \in \mathcal{M}$, which by definition belongs to the visibility set K. However, the trajectories $\boldsymbol{q}(t)$ do not necessarily belong to the visibility set K $\forall t \geq 0$, i.e. L may not be visible to F during some finite time interval. This mainly depends on the choice of $\boldsymbol{q_d} \in \mathcal{M}$, which dictates the vector $\boldsymbol{p}$ for the reference vector field (10), i.e. the reference orientation $\varphi(t)$ that the robot has to track via (12b).

Thus, given a $\boldsymbol{q}(0) \in K$, one should first select a $\boldsymbol{q_d} \in \mathcal{M}$ that ensures visibility maintenance under (12). In this respect, note that not all possible desired positions $\boldsymbol{r_d}$ belong to the cone-of-view at each t; see for instance Fig. 2: the desired positions $\boldsymbol{r_d} \in \mathbb{R}^2$ belong to the circle $c = \{\boldsymbol{r} \in \mathbb{R}^2 \mid x^2 + y^2 = {r_d}^2\}$, centered at the origin of $\mathcal{L}$; however, only the positions on the arc $\mathcal{V}$ shown in bold belong to the cone-of-view of F. Thus, it makes sense to pick some $\boldsymbol{r_d} \in \mathcal{V}$. Furthermore, out of the available options, it makes sense to pick the position $\boldsymbol{r_d} \in \mathcal{V}$ which lies on the line that connects the two robots. In this case, the orientation error to be regulated via the control input w_F is $e = \theta_\mathrm{F} - \varphi(x_1, y_1)$, where $\varphi(x_1, y_1) = \varphi_p = \theta_d$ (Fig. 2), while $\phi = $

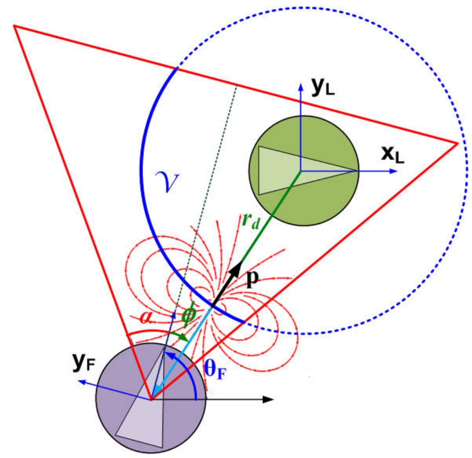


Fig. 2. Determining the vector $\boldsymbol{p}$ and the desired position $\boldsymbol{r_d}$ on $\mathbb{R}^2$

$\varphi_p - \theta_\mathrm{F} = -e$. Moreover, with this choice one has that $\phi \to \frac{1}{k_2} w_\mathrm{L}$, which further implies that k_2, w_L can be tuned so that $|\frac{1}{k_2} w_\mathrm{L}| \leq \alpha$.

In order to ensure that the system trajectories $\boldsymbol{q}(t)$ never escape the visibility set K, one has to consider how the system behaves on the boundary ∂K of K, where at least one of the constraints becomes active: $h_k(\boldsymbol{q}) = 0$ for some k. In particular, for $\boldsymbol{q} \in \partial K$ one has to check whether the system vector field $\dot{\boldsymbol{q}} = \boldsymbol{G}(\boldsymbol{q})\boldsymbol{v_\mathrm{F}}(\boldsymbol{q})$ is "tangent" to K, for bringing the solution $\boldsymbol{q}(t)$ back in the interior of K [8]. Thus, given that the constraints $h_k(\cdot) : \mathbb{R}^3 \to \mathbb{R}$ are continuously differentiable functions, one has thus to check whether

$$\dot{h}_k(\boldsymbol{q}) = \nabla h_k \dot{\boldsymbol{q}} = \nabla h_k \left(\boldsymbol{f}(\boldsymbol{q}, \boldsymbol{v_\mathrm{F}}) + \boldsymbol{g}(\boldsymbol{q}, \boldsymbol{v_\mathrm{L}})\right) < 0, \tag{13}$$

for all $\boldsymbol{q} \in \partial K$ where $h_k(\boldsymbol{q}) = 0$, for each k. If (13) holds, then the value of $h_k(\boldsymbol{q})$ is forced to decrease, bringing the trajectory $\boldsymbol{q}(t)$ into the visibility set K. In other words, visibility is maintained if and only if the condition (13) holds $\forall k$. Consequently, if (13) does not hold for some k, switching to a different control $\boldsymbol{v_\mathrm{F}}(\boldsymbol{q})$ that satisfies (13) should occur.

Similarly, the necessary conditions for maintaining visibility when *all* the constraints are active *at the same time* are written using the Jacobian matrix $\boldsymbol{J_h}(\boldsymbol{q})$ of the map $\boldsymbol{h} = (h_1(\cdot), h_2(\cdot)) : \mathbb{R}^3 \to \mathbb{R}^2$ as

$$\boldsymbol{J_h}(\boldsymbol{q})\dot{\boldsymbol{q}} < \mathbf{0}, \text{ where } \boldsymbol{J_h}(\boldsymbol{q}) = \begin{bmatrix} \frac{\partial h_1}{\partial x} & \frac{\partial h_1}{\partial y} & \frac{\partial h_1}{\partial \theta_\mathrm{F}} \\ \frac{\partial h_2}{\partial x} & \frac{\partial h_2}{\partial y} & \frac{\partial h_2}{\partial \theta_\mathrm{F}} \end{bmatrix}. \tag{14}$$

To illustrate this, consider the time derivative of $h_1(\cdot)$ for $\phi \geq 0$, that reads

$$\dot{h}_1 = \begin{bmatrix} -\frac{y}{x^2+y^2} & \frac{x}{x^2+y^2} & -1 \end{bmatrix} \begin{bmatrix} u_\mathrm{F}\cos\beta - u_\mathrm{L} + y w_\mathrm{L} \\ u_\mathrm{F}\sin\beta - x w_\mathrm{L} \\ w_\mathrm{F} - w_\mathrm{L} \end{bmatrix} =$$

$$= \frac{u_\mathrm{F}}{x^2+y^2}(x\sin\beta - y\cos\beta) - w_\mathrm{F} + \frac{y u_\mathrm{L}}{x^2+y^2}, \tag{15}$$

and the time derivative of $h_2(\cdot)$ for $\phi \geq 0$, that reads

$$\dot{h}_2 = \frac{u_\mathrm{F}}{\sqrt{x^2+y^2}} \left(\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} + \frac{L_S \cos\alpha \tan\phi}{\sqrt{x^2+y^2}\cos\phi} \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} -\sin\beta \\ \cos\beta \end{bmatrix} \right) +$$

$$+ \frac{L_S \cos\alpha \tan\phi}{\cos\phi} w_\mathrm{F} + \left| \frac{x}{\sqrt{x^2+y^2}} + \frac{y L_S \cos\alpha \tan\phi}{(x^2+y^2)\cos\phi} \right| u_\mathrm{L}. \tag{16}$$

Then, the control gains k_1, k_2 should be chosen such that the constraints are not violated in the worst-case scenario, where

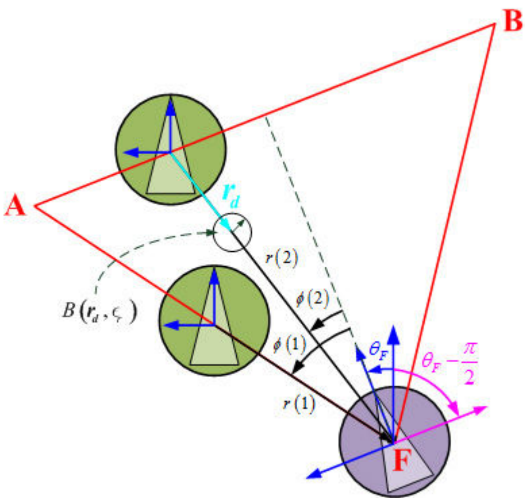


Fig. 3. In an obstacle-free environment, the viability constraints are nearly violated on the boundary of the cone of view (visibility at stake)

both constraints are active. This in turn yields the following sufficient condition:

$$A + \frac{u_{\mathrm{LM}} \sin \alpha}{L_s} \le w_{\mathrm{F}} \le A + \frac{u_{\mathrm{F}}}{{L_s}^2 \tan \alpha} \left| x \cos \beta + y \sin \beta \right| - \\ - \frac{u_{\mathrm{LM}}}{L_s \sin \alpha}, \quad \text{where } A = \frac{u_{\mathrm{F}}}{{L_s}^2} \left| x \sin \beta - y \cos \beta \right| .$$

For a more detailed treatment please consider [11].

Remark: The above discussion implies that if the control gains k_1, k_2 in (12) are such that $\dot{h}_k(\boldsymbol{q}) < 0$, $\forall \boldsymbol{q} \in \partial K$, $\forall k$, then F is guaranteed to maintain visibility w.r.t. L, and furthermore to converge and remain into the ball $\mathcal{B}(\boldsymbol{r_d}, \epsilon_r)$. The orientation control (via w_{F}) for F ensures that ϕ is exponentially stable to $-\frac{1}{k_2}|w_{\mathrm{L}}|$, which can be tuned to be $\le \alpha$, i.e. $h_1(\cdot)$ is always forced to be negative. This in turn implies that the system trajectories are always forced away from the boundary of K that corresponds to $h_1 = 0$. Furthermore, one has from the convergence analysis of the system trajectories into the ball $\mathcal{B}(\boldsymbol{r_d}, \epsilon_r)$ [11] that $\dot{V}_1$ is negative in the set $\{\boldsymbol{r_1} \mid \frac{1}{2}{\epsilon_r}^2 \le V_1(\|\boldsymbol{r_1}\|) \le \frac{1}{2}\|\boldsymbol{r_1}(0)\|^2\}$, where $V_1 = \frac{1}{2}({x_1}^2 + {y_1}^2)$, which implies that the distance $\|\boldsymbol{r_1}\|$ decreases under the control law u_{F}; this implies that $h_2(\cdot)$ is forced to decrease. These two conditions verify that if F starts somewhere in the interior of K, or on the boundary ∂K, it never reaches again the boundary ∂K on its way to $\boldsymbol{r_d}$. Finally, collision avoidance between the two robots is ensured since $\dot{V}_1$ is negative out of the ball $\mathcal{B}(\boldsymbol{r_d}, \epsilon_r)$.

IV. Motion Planning in Obstacle Environments

The $\mathrm{L} - \mathrm{F}$ formation is assumed to move in a structured workspace $\mathcal{W} \subset \mathbb{R}^2$ with known obstacles (e.g. an indoor corridor environment), where F is controlled by the control law (12). The motion of both robots is restricted due to the obstacles, and therefore the trajectories $\boldsymbol{q_{\mathrm{L}}}(t)$, $\boldsymbol{q_{\mathrm{F}}}(t)$ should be collision-free. Given that the robots can be represented as circular disks of radius r_0, the obstacles are inflated as shown in Fig. 4. Then, the dark grey region around each obstacle reduces the system free space, while it does not affect visibility; F can still detect L through this region, but

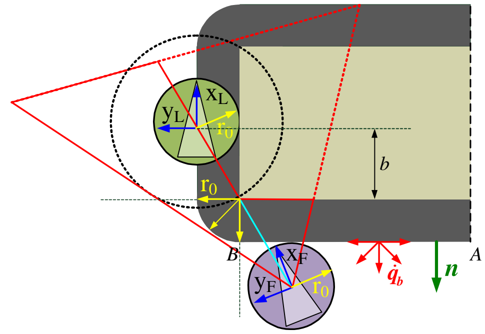


Fig. 4. In an obstacle environment, the "viability" constraints are active on the boundary of the cone of view (visibility at stake) and on the boundary of the inflated obstacles (safety due to collisions at stake)

both F, L should not enter into it. This requirement can be encoded as additional constraint inequalities, so that the same analysis on the boundary of the viability (safe) set can be applied, as for the visibility constraints.

In order to design a state feedback control scheme for an $\mathrm{L} - \mathrm{F}$ formation that has to move through a corridor environment, we first decompose the free space into rectangular cells. L is assigned a global high-level discrete motion planner, which indicates the successive order of cells that L has go through in order to converge to a goal state $\boldsymbol{q_{d\,\mathrm{L}}}$. Then, a dipolar vector field (10) of certain desired properties is defined into each one of the cells. The desired properties are specified by the motion plan: the vector field in a cell i is constructed so that its integral curves point into the interior of the successor cell $i+1$ on the exit face of the cell i, while pointing into the interior of the cell i on each one of the remaining faces.This approach is similar in spirit with the one in [12]. The difference is that the vector fields defined in each cell i are dipolar, so that the integral curves converge to the midpoint of the exit face of cell i.

The feedback plan for L is defined as to orient with and flow along the integral curves of the vector field in each cell. To do so, the control inputs for L are defined as

$$u_{\mathrm{L}} = \text{const} \le u_{\mathrm{L\,M}}, \tag{17a}$$

$$w_{\mathrm{L}} = -k_{\mathrm{L}}(\theta_{\mathrm{L}} - \varphi_{\mathrm{L}\,i}) + \dot{\varphi}_{\mathrm{L}\,i}, \tag{17b}$$

where $\varphi_{\mathrm{L}\,i}$ is the orientation of the vector field in cell i, $k_{\mathrm{L}} > 0$. Collision avoidance for L is ensured since by definition each vector field points into the interior of the free space. Furthermore, the trajectories of L essentially dictate the desired position $\boldsymbol{r_d}(t) \in \mathcal{V}$ (Fig. 2) that F has to track at each t; clearly $\boldsymbol{r_d}(t)$ should always lie in the free space. To see if this is always the case, let us first assume that L, F start in the same cell i, at initial distance $r > r_d$. Initial configurations that violate visibility are excluded. In the worst case, both robots start on the boundary of the free space. It is easy to verify that if the initial orientations θ_{L}, θ_{F} point into the interior of the cell i, then under the control laws (17), (12) both robots move into the cell i, and thus collisions with obstacles are avoided. Inter-robot collision avoidance is also guaranteed, as shown in Section III; therefore, their motion in cell i is guaranteed to be collision-free.

However, when L enters cell $i+1$ while F is still in cell i, it is likely that $\boldsymbol{q}_{\mathbf{L}}(t)$ will force the desired position $\boldsymbol{r_d}$ to eventually enter the obstacle space. This remark implies that L should move with a minimum turning radius R_{L} when entering into cell $i+1$, such that the trajectories $\boldsymbol{q}_{\mathbf{F}}(t)$ do not enter the obstacle space. Note also that after F has converged into $\mathcal{B}(\boldsymbol{r_d}, \epsilon_r)$, where $\epsilon_r \to 0$, the $\mathrm{L}-\mathrm{F}$ formation essentially behaves as a tractor (L) pulling a trailer (F) with axle-to-axle hitching of length r_d [13]. Then, if L starts moving along a circle of center C and radius R_{L} when it enters cell $i+1$, it immediately follows that F will move on a circle of the same center C and radius $R_{\mathrm{F}} = \sqrt{{R_{\mathrm{L}}}^2 - {r_d}^2}$.

In order to get an estimate for picking a safe R_{L}, consider Fig. 5: Assume that at time instant $t^\star$, L is at the midpoint of the cell i (driven there by (17)) and starts moving in cell $i+1$ along a circle of radius R_{L}, while F is at a distance r_d w.r.t. L; given that F kinematically behaves as a trailer, it starts moving along a circle of radius $R_{\mathrm{F}} = \sqrt{{R_{\mathrm{L}}}^2 - {r_d}^2}$. The radius R_{F} is depicted in Fig. 5 equal to the critical value $R_{\mathrm{F,crit}}$ for which the trajectories of F remain collision-free. The center of rotation C remains constant, and its position at time $t^\star$ w.r.t. the (time-varying) leader frame $\mathcal{L}$ is $x_C(t^\star) = x_{\mathrm{F}}(t^\star) + R_{\mathrm{F}} \sin(\beta(t^\star))$, $y_C(t^\star) = y_{\mathrm{F}}(t^\star) - R_{\mathrm{F}} \cos(\beta(t^\star))$. The coordinates of the critical point Z w.r.t. the leader frame $\mathcal{L}$ at time $t^\star$ are $x_{\mathrm{Z}}(t^\star) = \frac{r_0\sqrt{2}}{2} - r_0$, $y_{\mathrm{Z}}(t^\star) = -\frac{w_i}{2} + \frac{r_0\sqrt{2}}{2} - r_0$. Thus, $R_{\mathrm{F,crit}} = \sqrt{(x_C(t^\star) - x_{\mathrm{Z}}(t^\star))^2 + (y_C(t^\star) - y_{\mathrm{Z}}(t^\star))^2}$. Note also that the smallest critical value $R_{\mathrm{F,crit}}$ corresponds to the worst-case scenario shown in Fig. 5, where F is on the boundary of the free space.[1] At this point, one has $x_{\mathrm{F_w}}(t^\star) = -\sqrt{{r_d}^2 - \frac{{w_i}^2}{4}}$, $y_{\mathrm{F_w}}(t^\star) = -\frac{w_i}{2}$, $\cos(\beta_{\mathrm{w}}(t^\star)) = \frac{\sqrt{{r_d}^2 - \frac{{w_i}^2}{4}}}{r_d}$, $\sin(\beta_{\mathrm{w}}(t^\star)) = \frac{w_i}{2r_d}$. Then, after some algebra one can verify out of $R_{\mathrm{F_w,crit}} = R_{\mathrm{F_w}}$ that the worst-case safe $R_{\mathrm{F_w}}$ is

$$R_{\mathrm{F_w}} = \frac{\left(\frac{r_0}{2}\sqrt{2} - r_0 + \sqrt{{r_d}^2 - \frac{{w_i}^2}{4}}\right)^2 + \left(\frac{r_0}{2}\sqrt{2} - r_0\right)^2}{\frac{w_i}{r_d}\sqrt{{r_d}^2 - \frac{{w_i}^2}{4}} + \frac{w_i r_0}{r_d}\left(\frac{\sqrt{2}}{2} - 1\right) - \frac{2r_0}{r_d}\sqrt{{r_d}^2 - \frac{{w_i}^2}{4}}\left(\frac{\sqrt{2}}{2} - 1\right)}, \quad (18)$$

where the denominator is positive for $\frac{w_i}{r_d}\sqrt{{r_d}^2 - \frac{{w_i}^2}{4}} + \frac{2r_0}{r_d}\sqrt{{r_d}^2 - \frac{{w_i}^2}{4}}\left(1 - \frac{\sqrt{2}}{2}\right) > \frac{w_i r_0}{r_d}\left(1 - \frac{\sqrt{2}}{2}\right)$.

As one would expect from physical intuition, the critical turning radius for F depends on the robots' dimension r_0, the distance r_d between them and the width w_i of the cell i. Thus, if L moves with turning radius $R_{\mathrm{L}} \geq \sqrt{{r_d}^2 + {R_{\mathrm{F_w}}}^2}$, it follows that the trajectories of F are collision-free. Moreover, one can easily verify out of Fig. 5 that the motion of L within cell $i+1$ is collision-free as long as $R_{\mathrm{L}} \leq w_{i+1} + R_{\mathrm{F_w}} \frac{w_i}{2r_d} + \sqrt{{r_d}^2 - \frac{{w_i}^2}{4}}$. In summary, a safe R_{L} for L should satisfy

$$\sqrt{r_d^2 + {R_{\mathrm{F_w}}}^2} \leq R_{\mathrm{L}} \leq w_{i+1} + R_{\mathrm{F_w}} \frac{w_i}{2r_d} + \sqrt{r_d^2 - \frac{w_i^2}{4}}, \quad (19)$$

where $R_{\mathrm{F_w}}$ is given by (18), and ensures that the trajectories $\boldsymbol{q}_{\mathbf{L}}(t)$, $\boldsymbol{q}_{\mathbf{F}}(t)$ are collision-free. Note that this is a conservative condition, in the sense that $R_{\mathrm{F_w}}$ is computed for the worst-case scenario, since we assumed that L has no information on the position of F at time $t^\star$. Nevertheless, given r_d, r_0 and the cell decomposition, it is easy to a priori check whether a safe R_{L} exists for each one of the transitions between cells that are realized as turning around corners.

[1]This holds for the leader taking a right turn. Similarly one can treat the case for a left turn.

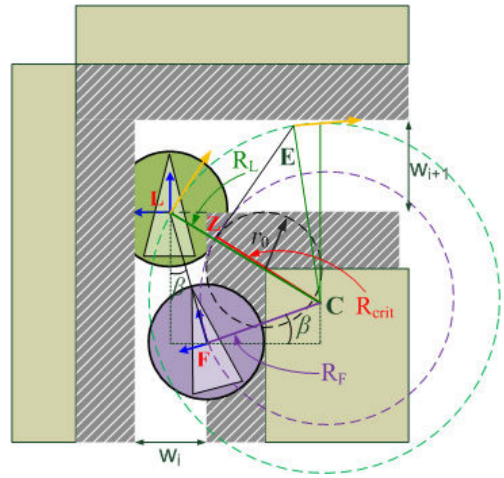


Fig. 5. After exiting cell i, the leader should move in cell $i+1$ along a circle of radius R_{L} that satisfies (19).

Given that a safe R_{L} exists, the control input (17) for L is modified as follows: On the exit face of cell i, L orients with the tangent λ_1 to the radius CL, given as $\lambda_1 = -\frac{x_C(t^\star)}{y_C(t^\star)} = -\frac{x_{\mathrm{F_w}}(t^\star) + R_{\mathrm{F_w}} \sin(\beta_{\mathrm{w}}(t^\star))}{y_{\mathrm{F_w}}(t^\star) - R_{\mathrm{F_w}} \cos(\beta_{\mathrm{w}}(t^\star))}$, and moves into cell $i+1$ with $w_{\mathrm{L\,c}} = \mathrm{sign}(w_{\mathrm{L}}) \frac{u_{\mathrm{L}}}{R_{\mathrm{L}}}$, where w_{L} is the angular velocity that is dictated by the vector field in cell $i+1$. The angular velocity $w_{\mathrm{L\,c}}$ should be kept until L reaches the point E shown in Fig. 5, so that F is guaranteed to enter safely in the cell $i+1$. The coordinates of E w.r.t. the initial frame at time $t^\star$ are $x_{\mathrm{E}} = x_C + R_{\mathrm{L}} \cos\theta_{\mathrm{L}}$, $y_{\mathrm{E}} = y_C - R_{\mathrm{L}} \sin\theta_{\mathrm{L}}$, where θ_{L} is the current orientation of L w.r.t. a global frame, which is online available. After L has reached E, then both robots are in cell $i+1$, with F pointing into the interior of the free space. Thus, L keeps moving under (17), tracking the vector field in cell $i+1$, on its way to the exit face of this cell; as discussed above, under these conditions the motion of the robots within the cell $i+1$ is collision-free.

A. Simulation Results

The efficacy of the proposed motion planning and control scheme has been evaluated through computer simulations. A decomposition of the free space in rectangular cells is known to L. The robots start in the same cell i, so that L is visible to F. In the case shown in Fig. 6 the robots initiate on the boundary of the obstacle, so that the second visibility constraint is active for F and so that they do not face out of the free space. The leader L moves with constant linear velocity $u_{\mathrm{L}} \leq u_{\mathrm{L\,M}}$, and tracks the vector field in cell i under (17), on its way to the midpoint of the exit face of i. At the same time, F moves under the control law (12), where the control gains are selected such that the visibility constraint $h_2(\boldsymbol{q}(0)) = 0$ is not violated at $t = 0$, and converges into a neighborhood around the desired configuration $\boldsymbol{q_d}$ (the red mark) w.r.t. L. The motion of F in cell i is also collision-

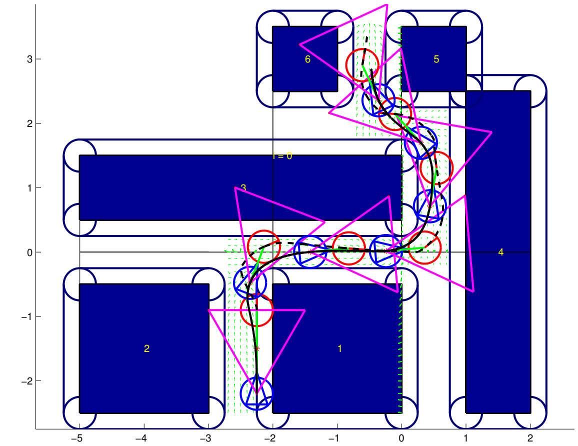
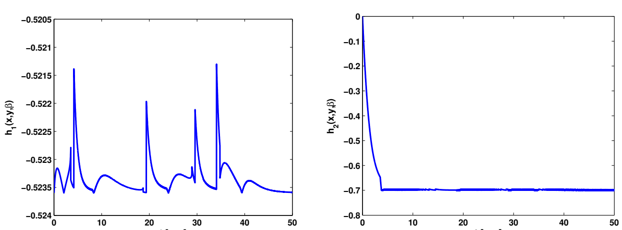


Fig. 6. The system initiates on a configuration $\boldsymbol{q} \in \mathcal{C}$ on the boundary of the obstacles, where the second visibility constraint is active for F.

free, for the reasons explained in section III. When L reaches the exit face of cell i, it is forced to follow a bounded curvature within cell $i+1$ to move around the corner, under the angular velocity $w_{\mathrm{L\,c}} = \mathrm{sign}(w_{\mathrm{L}})\frac{u_{\mathrm{L}}}{R_{\mathrm{L}}}$, where w_{L} is the angular velocity specified by the vector field in cell $i+1$, and R_{L} is the safe turning radius calculated as above. At the same time, F behaves like a trailer and starts moving along a circle of radius R_{F}, which is guaranteed to be collision-free. As soon as L reaches the "exit point" E, shown in Fig. 5, it continues moving under the angular velocity w_{L} dictated by the vector field $i+1$, until it reaches the exit face of cell $i+1$, and so on. The resulting trajectories are collision-free, as shown in Fig. 6, while the value of the constraints $h_k(x, y, \beta)$, $k = 1, 2$ is always non-positive (Fig. 7), which implies that visibility is always maintained.



(a) $h_1(x, y, \beta) \leq 0$, $\forall t$.

(b) $h_2(x, y, \beta) \leq 0$, $\forall t$.

Fig. 7. Visibility is always maintained, since the value of the visibility constraints is always negative.

V. Discussion

This paper presented a feedback control solution for an L − F formation with visibility constraints in an environment with obstacles. A control scheme that forces F to converge and remain into a set of desired configurations w.r.t. L without violating visibility was proposed, as well as a way of controlling L in a known obstacle environment, so that both obstacle avoidance and visibility maintenance are ensured. The proposed control schemes are decentralized, since there is no direct communication between the robots. Computer simulations demonstrate the efficacy of our algorithms.

Finally, it is worth to mention that the proposed control design ideas are not restricted to the scenarios presented in this paper. Given a cell decomposition of convex polygonal cells, L can be controlled to move from cell i to cell $i+1$ by tracking a dipolar vector field in cell i, defined so that its integral curves converge to the midpoint of the exit face of cell i, while pointing into the interior of the cell i on the remaining faces. For ensuring the collision-free motion of F as it moves from cell i to cell $i+1$, one can similarly employ the tractor-trailer paradigm, and pick a safe turning radius R_{L} for L by computing the worst-case safe turning radius $R_{\mathrm{F_w}}$ for F. The tractor-trailer paradigm can be also used to extend the formation control in the case of N > 2 robots that move in a chain formation, in the sense that if the tractor (L) moves along a circle of center C and radius R_{L}, then the N − 1 trailers (the followers F_j, $j = 1, \ldots, \mathrm{N}-1$) move along circles of center C and radii R_{F_j}; thus, one can compute a (conservative) condition on a safe R_{L} so that the (worst-case) turning radius R_{F_j} for the j-th follower, $j = 1 \ldots, \mathrm{N}-1$, is collision-free. Finally, the assumption on the sensor footprint being an isosceles triangle is not restrictive, since the conditions on maintaining visibility apply to any closed convex footprint.

References

[1] A. K. Das, R. Fierro, V. Kumar, J. P. Ostrowski, J. Spletzer, and C. J. Taylor, "A vision-based formation control framework," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 5, pp. 813–825, Oct. 2002.

[2] N. Cowan, O. Shakernia, R. Vidal, and S. Sastry, "Vision-based follow-the-leader," in *Proc. of the 2003 IEEE/RSJ Intl. Conference on Intelligent Robots and Systems*, Las Vegas, Nevada, Oct. 2003, pp. 1796–1801.

[3] G. L. Mariottini, F. Morbidi, D. Prattichizzo, N. Vander Valk, G. Pappas, and K. Daniilidis, "Vision-based localization for leader-follower formation control," *IEEE Transactions on Robotics*, vol. 25, no. 6, pp. 1431–1438, Dec. 2009.

[4] G. Kantor and A. A. Rizzi, "Feedback control of underactuated systems via sequential composition: Visually guided control of a unicycle," in *Robotics Research*, ser. Springer Tracts in Advanced Robotics, P. Dario and R. Chatila, Eds. Springer Berlin Heidelberg, 2005, pp. 281–290.

[5] S. Bhattacharya, R. Murrieta-Cid, and S. Hutchinson, "Optimal paths for landmark-based navigation by differential-drive vehicles with field-of-view constraints," *IEEE Transactions on Robotics*, vol. 23, no. 1, pp. 47–59, Feb. 2007.

[6] P. Salaris, D. Fontanelli, L. Pallotino, and A. Bicchi, "Shortest paths for a robot with nonholonomic and field-of view constraints," *IEEE Transactions on Robotics*, vol. 26, no. 2, pp. 269–280, Apr. 2010.

[7] F. Morbidi, F. Bullo, and D. Prattichizzo, "Visibility maintenance via controlled invariance for leader-follower vehicle formations," *Automatica*, vol. 47, no. 5, pp. 1060–1067, May 2011.

[8] J.-P. Aubin, *Viability Theory*. Birkhauser, 1991.

[9] H. K. Khalil, *Nonlinear Systems. Third Edition*. Prentice-Hall, 2002.

[10] D. Panagou, H. G. Tanner, and K. J. Kyriakopoulos, "Dipole-like fields for stabilization of systems with Pfaffian constraints," in *Proc. of the 2010 IEEE International Conference on Robotics and Automation*, Anchorage, Alaska, May 2010, pp. 4499–4504.

[11] D. Panagou and V. Kumar, "Maintaining visibility for leader-follower formations in obstacle environments," Technical Report, available at http://users.ntua.gr/dpanagou/techrep.pdf.

[12] S. R. Lindemann and S. M. LaValle, "Simple and efficient algorithms for computing smooth, collision-free feedback laws over given cell decompositions," *International Journal of Robotics Research*, vol. 28, no. 5, pp. 600–621, 2009.

[13] L. Bushnell, B. Mirtich, A. Sahai, and M. Secor, "Off-tracking bounds for a car pulling trailers with kingpin hitching," in *Proc. of the 33th IEEE Conference on Decision and Control*, Lake Buena Vista, FL, USA, Dec. 1994, pp. 2944–2949.