Atomic Gas Density Profile

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November 7, 2012

1 Thomas-Fermi Profile

A Bose-Einstein condensate can be well described in the bulk by a *Thomas-Fermi density profile* [1]

$$n_{3D}(x,y,z) = n_0\left[1-\left(\frac{x}{\sigma_x}\right)^2-\left(\frac{y}{\sigma_y}\right)^2-\left(\frac{z}{\sigma_z}\right)^2\right] \tag{1}$$

where n_0 is the peak density.

The 2D atomic density in the XY plane is obtained by integrating $n(x,y,z)$ along $\hat{z}$, i.e. $n(x,y) = \int dz\ n(x,y,z)$. The integration limits are set by the condition $n(x,y,z)=0$, resulting in $\pm z_0$, where $z_0 \equiv \sigma_z\sqrt{1-(x/\sigma_x)^2-(y/\sigma_y)^2}$

$$\begin{aligned} n_{2D}(x,y) &= 2n_0\left[\left(1-\left(\frac{x}{\sigma_x}\right)^2-\left(\frac{y}{\sigma_y}\right)^2\right)z-\frac{z^3}{3\sigma_z^2}\right]_0^{z_0} && (2)\\ &= \frac{4}{3}n_0\sigma_z\left[1-\left(\frac{x}{\sigma_x}\right)^2-\left(\frac{y}{\sigma_y}\right)^2\right]^{3/2} && (3)\end{aligned}$$

Analogously, the 1D atomic density in $\hat{x}$ is given by $n(x) = \int dy\ n(x,y)$, and the integration limits are $\pm y_0$, where $y_0 \equiv \sigma_y \sqrt{1-(x/\sigma_x)^2}$

$$n_{1D}(x) = \frac{8 n_0 \sigma_z}{3\sigma_y^3} \underbrace{\int_0^{y_0} dy \left(y_0^2 - y^2\right)^{3/2}}_{\frac{3\pi}{8} y_0^4} \tag{4}$$

$$= \frac{\pi}{2} n_0 \sigma_y \sigma_z \left[1 - \left(\frac{x}{\sigma_x}\right)^2\right]^2 \tag{5}$$

The total atom number N is

$$N = \int_{-\sigma_x}^{\sigma_x} dx \; n(x) \tag{6}$$

$$= \frac{\pi}{2} n_0 \sigma_y \sigma_z \underbrace{\int_{-\sigma_x}^{\sigma_x} dx \left[1 - \left(\frac{x}{\sigma_x}\right)^2\right]^2}_{\frac{16}{8}\sigma_x} \tag{7}$$

$$= \frac{8}{15} \pi n_0 \sigma_x \sigma_y \sigma_z \tag{8}$$

Given that the volume enclosed by the ellipsoid $(x/\sigma_x)^2 - (y/\sigma_y)^2 - (z/\sigma_z)^2 = 1$ is $V = \int dxdydz = \frac{4\pi}{3}\sigma_x\sigma_y\sigma_z$, then Eq.(8) can be written as

$$\boxed{N = \frac{2}{5} n_0 V} \tag{9}$$

In other words, the average density $\bar{n} \equiv N/V$ is 2/5 of the peak density n_0.

n_{3D}, n_{2D}, and n_{1D} can also be written in terms of N

$$n_{3D}(x,y,z) = \frac{15}{8\pi} \frac{N}{\sigma_x\sigma_y\sigma_z} \left[1 - \left(\frac{x}{\sigma_x}\right)^2 - \left(\frac{y}{\sigma_y}\right)^2 - \left(\frac{z}{\sigma_z}\right)^2\right] \tag{10}$$

$$n_{2D}(x,y) = \frac{5}{2\pi} \frac{N}{\sigma_x\sigma_y} \left[1 - \left(\frac{x}{\sigma_x}\right)^2 - \left(\frac{y}{\sigma_y}\right)^2\right]^{3/2} \tag{11}$$

$$n_{1D}(x) = \frac{15}{16} \frac{N}{\sigma_x} \left[1 - \left(\frac{x}{\sigma_x}\right)^2\right]^2 \tag{12}$$

2 Thermal Gas Profile

A thermal gas is decribed by the Maxwell-Boltzmann distribution

$$n_{3D}\left(v_x, v_y, v_z\right) = N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}\left(v_x^2+v_y^2+v_z^2\right)} \tag{13}$$

The atomic density in the XY plane is

$$n_{2D}\left(v_x, v_y\right) = N\frac{m}{2\pi kT} e^{-\frac{m}{2kT}\left(v_x^2+v_y^2\right)} \tag{14}$$

In absorption imaging, we fit the thermal gas density to a gaussian model

$$n_{2D}\left(v_x, v_y\right) = n_0 e^{-\frac{1}{2}\left[\left(\frac{v_x}{\sigma_x}\right)^2+\left(\frac{v_y}{\sigma_y}\right)^2\right]} \tag{15}$$

We can connect the fit parameters to physical quantities by identification

$$n_0 = Nm/2\pi kT \quad \sigma_x = \sqrt{kT/m} \quad \sigma_y = \sqrt{kT/m} \tag{16}$$

Notice that $\sigma_x = \sigma_y$ for a thermal gas

3 Question

Because of resolution constraint, a standard technique for imaging an atomic gas is the so-called *time-of-flight* method, where the gas is released from the trap and allowed to expand. For a non-interacting system, the only force acting on the system is gravity. The gas evolution in time is determined by the propagator (classical or quantum). A gaussian packet is still gaussian after tof, but what about a TF profile? why is n_{2D} still a good fit after time of flight?

References

[1] Pethick and Smith, Bose-Einstein Condensation in Dilute Gases