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Hemispheric asymmetries in processing numerical meaning in arithmetic

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A B S T R A C T

Hemispheric asymmetries in arithmetic have been hypothesized based on neuropsychological, developmental, and neuroimaging work. However, it has been challenging to separate asymmetries related to arithmetic specifically, from those associated general cognitive or linguistic processes. Here we attempt to experimentally isolate the processing of numerical meaning in arithmetic problems from language and memory retrieval by employing novel non-symbolic addition problems, where participants estimated the sum of two dot arrays and judged whether a probe dot array was the correct sum of the first two arrays. Furthermore, we experimentally manipulated which hemisphere receive the probe array first using a visual half-field paradigm while recording event-related potentials (ERP). We find that neural sensitivity to numerical meaning in arithmetic arises under left but not right visual field presentation during early and middle portions of the late positive complex (LPC, 400-800 ms). Furthermore, we find that subsequent accuracy for judgements of whether the probe is the correct sum is better under right visual field presentation than left, suggesting a left hemisphere advantage for integrating information for categorization or decision making related to arithmetic. Finally, neural signatures of operational momentum, or differential sensitivity to whether the probe was greater or less than the sum, occurred at a later portion of the LPC (800-1000 ms) and regardless of visual field of presentation, suggesting a temporal and functional dissociation between magnitude and ordinal processing in arithmetic. Together these results provide novel evidence for differences in timing and hemispheric lateralization for several cognitive processes involved in arithmetic thinking.

Arithmetic is typically learned in the first years of formal schooling and continually practiced in both formal and informal settings throughout the lifespan. Basic concepts of arithmetic form a foundation for higher order mathematical thinking and arithmetic ability is predictive of later mathematics achievement (Cantlon et al., 2009; Dehaene, 1997; Dehaene et al., 2004; Gallistel and Gelman, 1992; Gilmore, McCarthy, & Spelke, 2007, 2010). Given its foundational importance and ubiquitous use in modern life, there has been a recent surge in interest in how arithmetic is carried out in the brain (Ansari, 2008; Arsalidou and Tayler, 2011; Dehaene, 1997; Dehaene and Cohen, 1995; Dehaene et al., 2004; Dehaene et al., 2003; Venkatraman et al., 2005). What is clear from this work is that arithmetic involves the engagement of many different perceptual, cognitive, and linguistic processes and their associated brain systems. This includes a network of regions thought to be specific to mathematical thinking (Dehaene et al., 1999), as well as the other perceptual, cognitive, linguistic, and memory systems that subserve arithmetic (Rosenberg-Lee et al., 2011; Avancini et al., 2014; Avancini et al., 2015). The particular combination and the extent of engagement of the various cognitive and brain processes appears to depend on experimental context, arithmetic operation, and problem-solving strategy taken by the participant (e.g., Cho et al., 2011; Grabner et al., 2009; Sohn et al., 2004; Zago et al., 2001). One question that remains is whether the left and right hemispheres contribute differentially to arithmetic. Some evidence suggests that there may be differential hemispheric contributions to arithmetic (Ansari, 2010; Ansari and Dhital, 2006; Cantlon et al., 2006; Chassy and Grodd, 2012; Cohen Kadosh et al., 2007; Dehaene, 1996; Emerson and Cantlon, 2015; Holloway and Ansari, 2010; Izard et al., 2008; Menon et al., 2000; Pinel et al., 2001; Rivera et al., 2005; Rosenberg-Lee et al., 2011; Hyde et al., 2010, Edwards et al., 2016, Libertus et al., 2009; Piazza et al., 2004; Piazza et al., 2007), but the known complexity and context-dependency of arithmetic has made it challenging to delineate them (e.g., Chochon et al., 1999; Lee, 2000; Dehaene and Cohen, 1997; Dehaene et al., 2003; Delazer and Benke, 1997; De Smedt, Holloway and Ansari, 2011; Prado2014; Prado et al., 2011; Rosenberg-Lee et al., 2011; Yi-Rong et al., 2011; Yu et al., 2011; Zhou et al., 2007). More specifically, it has

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been challenging to separate hemispheric organization of brain systems underlying mathematics-specific processes from the organization of other associated perceptual, cognitive, and linguistic processes subserving arithmetic that have their own hemispheric lateralization signatures (Dickson and Federmeier, 2017; Dehaene et al., 2003; Hyde et al., 2010). Here we attempt to provide new insight into hemispheric contributions to arithmetic by presenting addition problems non-symbolically and quickly, reducing the influence of language and memory retrieval processes (Barth et al., 2005; Barth et al., 2006; McCrink et al., 2007; Pica et al., 2004), with a visual half-field presentation technique that allows us to manipulate which hemisphere receives stimuli first (see Banich, 2003). While doing so, we measured the resulting time-resolved brain response via event-related potentials (ERPs). Under these experimental conditions, we ask if numerical meaning in arithmetic is processed equally by the left and right hemisphere or if there are functional asymmetries.

Arithmetic tasks reliably engage a variety of brain regions associated with various cognitive processes and many of these processes occur in parallel (see Rosenberg-Lee et al., 2011; Avancini et al., 2014, 2015). These include mathematics-specific (Amalric and Dehaene, 2016, 2018; 2019; Andres et al., 2011; Dehaene et al., 1999, 2003, 2004; Dehaene, 1997; Kawashima et al., 2004; Kucian et al., 2006; Piazza and Izard, 2009; Piazza et al., 2004, 2007; Rotzer et al., 2009; Venkatraman et al., 2005), linguistic-semantic (Baldo and Dronkers, 2007; Dehaene et al., 1999, 2003; Grabner and De Smedt, 2011; Gruber et al., 2001; Stoianov et al., 2004), memory (Grabner et al., 2009; Ischebeck et al., 2009; Peters and De Smedt, 2018), and other general cognitive processes (Metcalfe et al., 2013; Tschentscher& Hauk, 2014; Zago et al., 2001). Furthermore, the experimental context and problem-solving strategy taken by participants influences which processes and associated brain regions are activated (Cho et al., 2011; Sohn et al., 2004; Zago et al., 2001). For example, verbally-demanding story problems are associated with greater engagement of anterior prefrontal regions known to underlie language processing, whereas the same problems in equation form are associated with greater posterior parietal activation thought to underlie spatial attention and some mathematics-specific functions (Sohn et al., 2004). As another example, problems on which participants report using memory retrieval strategies are associated with stronger left angular gyrus activation, whereas problems on which participants report procedural strategies are associated with more widespread fronto-parietal network activity (Grabner et al., 2009). Relatedly, different arithmetic operations draw on different cognitive processes and associated brain systems (Chochon et al., 1999; Lee, 2000; Dehaene et al., 2003; De Smedt et al., 2011; Prado et al., 2011, 2014; Rosenberg-Lee et al., 2011; Yi-Rong et al., 2011; Yu et al., 2011; Zhou et al., 2007). For example, exact calculations are associated with greater activation in inferior frontal regions typically associated with language processing, whereas approximate calculations are associated with posterior parietal activation (e.g., Dehaene et al., 1999). As another example, operations like subtraction that draw more heavily on calculation are associated with greater posterior parietal activation, whereas operations like multiplication or simple addition that draw more on memory retrieval are associated with greater left-temporal parietal cortical activation (e.g., Prado et al., 2011). Neuropsychological studies also support the idea that different arithmetic operations drawn on different brain systems, as several cases show that lesions to parietal cortex result in impaired subtraction, but spared multiplication (Dehaene and Cohen, 1997; Delazer and Benke, 1997).

The involvement of many different cognitive processes combined with the level of context dependency of the engagement of these processes has made it challenging to understand hemispheric contributions. According to the predominate cognitive model in the field, the triple-code model, numerical processing involves engagement of at least three distinct neural codes or levels of number representation: a quantitative code, a verbal code, and visual code (Dehaene, 1992; Dehaene and Cohen, 1991; Dehaene et al., 2003).

The quantitative code that underlies the processing and manipulation of numerical meaning is thought to be carried out by bilateral regions of the parietal lobe (see Dehaene et al., 2003, see also Butterworth, 1999). Bilateral regions of the intraparietal sulcus respond reliability in a variety of numerical and mathematical tasks ranging from basic numerical comparison in older children and adults to truth evaluation of very high level mathematical formulas in professional mathematicians (Amalric and Dehaene, 2016, 2018; 2019; Cantlon et al., 2006, 2009; Pinel et al., 2001). At least some portion of intraparietal cortex is thought to be number-specific, meaning it engages selectively for numerical processing, but not other types of non-numerical processing (e.g., Piazza et al., 2004). Furthermore, bilateral IPS activates more when engaged in arithmetic problems that involve mental calculation or numerical manipulation (e.g., subtraction) compared to arithmetic problems that are commonly solved through memory retrieval (e.g., multiplication) (e.g., Chochon et al., 1999; Lee, 2000; Ischebeck et al., 2009; Prado et al., 2011, 2014; Polspoel et al., 2017). Thus, one straightforward prediction from the previous literature on mathematical processing in the brain is that processing of numerical meaning in arithmetic relies on bilateral regions of the parietal cortex.

To the extent that there are hemispheric differences in processing numerical meaning, those differences appear to be largely in favor of a right hemisphere advantage. First, when differences between the hemispheres are found in neuroimaging studies of processing numerical and mathematical meaning, activations are often stronger in the right compared to the left hemisphere (e.g., Cantlon et al., 2006; Chassy and Grodd, 2012; Cohen Kadosh et al., 2007; Dehaene, 1996; Holloway and Ansari, 2010; Holloway et al., 2010; Menon et al., 2000; Pinel et al., 2001). For example, when participants are asked to compare number words or digits, bilateral intraparietal activity is observed, but modulation of activity by numerical meaning is more prominent in the right compared to the left hemisphere (Pinel et al., 2001). As another example, contexts that require more calculation show greater activation compared to equally challenging conditions that require less calculation (e.g., Menon et al., 2000). Second, processing of numerical information appears to start out right-lateralized early in development and continues to show this asymmetry into childhood (Emerson and Cantlon, 2015; Izard et al., 2008; Hyde et al., 2010; Edwards et al., 2016; Libertus et al., 2009). Functional brain responses to numerical quantities and symbols become more bilateral over childhood, with left-lateralized activity increasing with both age and skill (Ansari and Dhital, 2006; Cantlon et al., 2006; Emerson and Cantlon, 2015; Kersey and Cantlon, 2017; Rivera et al., 2005; Rosenberg-Lee et al., 2011; Piazza et al., 2004, 2007). Together, this work suggests that the right hemisphere forms the initial basis for numerical thought, including basic arithmetic processing, whereas the left hemisphere becomes involved as older children learn the language and symbols associated with arithmetic. These additional lines of evidence suggest an alternative theory and a revision to the triple-code model: that processing of numerical meaning in arithmetic may be advantaged in the right hemisphere.

A compounding factor in a majority of the work on arithmetic in the brain is that most studies involve human adults contemplating relatively simple symbolic arithmetic problems for which they have a lifetime of experience. This raises a number of challenges for understanding the nature of arithmetic in the brain and distinguishing it from other cognitive and linguistic processes. The first challenge is that the answers to many of the types of arithmetic problems used in these studies can be solved by retrieving memorized facts. For example, most educated adults do not need to do any arithmetic to determine the sum of 5+5 or the product of 5×5 and, instead, just retrieve a fact from memory. Similarly, even in the case where the full answer is not already memorized, various strategies involving memory retrieval are likely to be engaged to derive an answer (e.g., columnar retrieval strategies for multiple-digit arithmetic such as $22+34 = (20+30) + (2+4)$) (Ashkenazi et al., 2014; Duverne and Lemaire, 2005; Lemaire and Arnaud, 2008). Some types of arithmetic like multiplication are often learned and practiced almost exclusively through memorization and fact retrieval. We certainly acknowledge that retrieving facts from memory is an effective strategy for solving arithmetic problems quickly. Furthermore, speed of retrieval is often used as metric of arithmetic mastery (i.e., mathematics fluency; Woodcock et al., 2001). However, in terms of understanding functional organization for arithmetic, brain correlates of arithmetic-specific processes should be delineated from general fact-retrieval. The use of familiar, often-memorized, symbolic arithmetic problems makes it challenging to separate the two.

The second challenge is that the particular symbolic numbers that make up common symbolic arithmetic problems used in research are riddled with semantic associations that are not likely to be essential to doing arithmetic but nonetheless influence processing. For example, the parity or decade of operands are known to influence strategy choice (see Domahs et al., 2007 for a discussion; Lemaire and Siegler, 1995; Lochy et al., 2000). Similarly, highly practiced symbolic numbers are intrinsically linked to language and other semantic memory processes, which are known to engage district, left-lateralized brain systems on their own (Boles, 1986; Cantlon and Brannon, 2007; Dehaene and Cohen, 1995; Dehaene et al., 2003; Salillas and Carreilas, 2014). Of course, it is well accepted that the left hemisphere is asymmetrically involved in verbal-linguistic processing (Federmeier, 2007; Risse et al., 1997; Gazzaniga and Sperry, 1967; Fromkin et al., 1974; McAdam and Whitaker, 1971; Kimura and Folb, 1968; Studdert-Kennedy & Shankweiler, 1970; Zatorre et al., 1992). Early neuropsychological studies showed that brain injury to the left hemisphere in some patients led to greater impairment in arithmetic than right hemisphere damage (Cipolotti et al., 1991; Dehaene and Cohen, 1997; Gerstmann, 1940; Rosselli and Ardila, 1989). Furthermore, the left hemisphere is often more strongly engaged than the right in simple, single digit symbolic addition compared to subtraction (Cohen et al., 2000; Dehaene and Cohen, 1997; De Jong, Van Zomeren, Willemsen and Paans, 1996; Pesenti et al., 2000; van Harskamp and Cipolotti, 2001). However, single-digit addition, at least in educated adults, relies more heavily on the retrieval of verbally-encoded answers from memory compared to subtraction. Left-lateralized biases in arithmetic, then, may be due to the intrinsic correlations between arithmetic and verbally-encoded numerical or arithmetic facts (e.g., $2 \times 2 = 4$) (Dehaene & Cohen, 1991, 1997; Grabner et al., 2009; Ischebeck et al., 2009; Lee, 2000; Zago et al., 2001). In fact, the triple-code model posits that the phonological, syntactic, and lexical aspects of numerals and arithmetic are processed in similar left-lateralized areas as those classically involved in linguistic processing of words and sentences, including left angular gyrus and portions of left prefrontal cortex (Dehaene et al., 1999, 2003; Piazza and Dehaene, 2004; Prado et al., 2011, 2014; Venkatraman et al., 2005). Thus, using stimuli that have such strong associative links to language and symbols makes it challenging to distinguish the brain systems engaged in arithmetic specifically from other verbal, semantic, and cognitive processes engaged through automatic association that are not critical to arithmetic itself.

Here we aimed to isolate processing of numerical meaning in arithmetic from other associated general cognitive, memory, and linguistic processes. To do this, we used a non-symbolic addition task, where participants were asked to estimate the numerical sum of two dot arrays and verify whether their estimate matched in number to a third probe array (see Barth et al., 2005, 2006; McCrink et al., 2007; Pica et al., 2004). Arrays of objects were generated for this experiment and presented rapidly so that the numerical values of the addends and, by consequence, the sum of those addends had to be quickly estimated and could not have been retrieved from memory (e.g., Barth et al., 2005). The choice to use arrays of objects also reduced the immediate associations with language that would be present with the use of numerical symbols. Other continuous, non-numerical magnitude parameters of the addends (i.e., dot arrays) were systematically varied using well-established controls (Hyde and Spelke, 2009; Hyde et al., 2014; Hyde and Wood, 2011; Khanum et al., 2016; Piazza et al., 2004) so that they were not predictive of the actual sum or outcome arrays across trials or the experiment as a whole.

We systematically manipulated the semantic-numerical relationship between the probe array (outcome) and the actual sum (Dehaene, 1996; Libertus et al., 2007; Pinel et al., 2001; Temple and Posner, 1998) by three levels. The probe contained the actual sum on one third of the trials (probe = actual sum), a numerically close but incorrect probe on another third of the trials (incorrect close), and a numerically distant and incorrect probe on a third of the trials (incorrect far). If the sum is being computed and numerically compared to the outcome array, then the brain response should scale with the numerical relationship between the numerical value of the outcome array presented and the sum computed and represented in the mind (see Avancini et al., 2014, 2015; Barth et al., 2005, 2006; Dehaene, 1996; Libertus et al., 2007; Pinel et al., 2001; Temple and Posner, 1998). In contrast, if brain sensitivities reflect simpler change detection responses or participants are not mentally calculating a mental sum at all (and therefore develop no numerical expectations), a similar response should be seen to all outcome arrays because all are novel with respect to the numerical and non-numerical magnitude parameters as well as other low-level visual parameters relative to the addends presented immediately beforehand. That is, there should be no differences in ERP modulation by the different numerical distances of the probe from the true sum.

The two incorrect distance levels (Incorrect Close and Incorrect Far) were used to further distinguish responses to numerical meaning from categorical correct vs. incorrect responses. Scaling of the response to incorrect outcomes based on their numerical distance from the sum would indicate processing of numerical meaning and rule out an interpretation of the response as a general categorical response to all outcomes judged as incorrect.

In order to test the roles of each hemisphere in processing numerical meaning in arithmetic, we systematically presented arithmetic verification probes to the left (LVF) and right visual field (RVF) using a visual half-field design as we recorded scalp electroencephalography (EEG) (Banich, 2003; Federmeier, 2007). The logic of this sort of paradigm is that presenting target stimuli to a lateralized visual field projects the information to a single hemisphere first, thereby providing a temporal advantage to the hemisphere contralateral to the visual field of presentation, even if information may eventually be communicated across the corpus collosum (see Banich, 2003). Visual half-field designs of this nature have been used extensively in concert with scalp electrophysiology measures (event-related potentials, ERP) in the domain of language processing to examine how each hemisphere processes word or sentence-level context information (e.g., Coulson et al., 2005; Federmeier and Kutas, 1999; Kemmer et al., 2014; Wlotko and Federmeier, 2007). Even if differences in the ERP response permeate widely across both hemispheres of the scalp, as is often the case in scalp ERPs, contrasting left with right visual field of presentation can reveal differences in function between hemispheres for the task at hand (Berardi and Florentini, 1997; Federmeier, 2007).

While several studies have examined the ERP correlates of arithmetic (e.g., Domahs et al., 2007; Jasinski and Coch, 2012; Jost et al., 2004a,b; Jost et al., 2004a,b; Niedeggen and Rösler, 1999), only one study to our knowledge has tested hemispheric contributions to arithmetic using the visual half-field paradigm with ERPs (Dickson and Federmeier, 2017). This study recorded ERPs in a (symbolic) multiplication problem verification task with adults. They reported two main effects: an earlier sensitivity of categorizing the probe as correct or not on the P3 (specifically P3b) irrespective of visual field of presentation and a later sensitivity to the semantic relatedness of the incorrect probes to the correct answer on the Late Positive Component/Complex (LPC) over the widespread central, parietal, and occipital electrode sites under left visual field presentation only (i.e., presentation to the right hemisphere). The P3b is well-studied component related to explicit recognition, discrimination, and categorization in decision making (e.g., Donchin and Coles, 1988; Smith and Guster, 1993). LPC is a later, slow-going positive composite of components related to further semantic processing, recognition, or decision confidence (e.g., Finnigan et al., 2002; Rubin et al., 1999). Thus, this study lends further support for the idea that there is an asymmetrical, right-hemisphere advantage in processing numerical meaning in arithmetic. Furthermore, the modulated components are generally consistent in timing and topography from what has been seen in other EEG/ERP studies of arithmetic (e.g., Domahs et al., 2007; Jost et al., 2004a,b; Niedeggen and Rösler, 1999). However, as described above, the use of symbolic multiplication raises the concern that the asymmetry observed might reflect hemispheric differences in how the brain retrieves a fact from memory or other associated semantic and linguistic processes rather than how numerical meaning is derived from the arithmetic process specifically. Here we use the same visual half-field paradigm and focus our analysis on the same ERP components to test for categorization of correctness and processing of semantic relatedness using three different distance conditions as in the previous related ERP study (Dickson and Federmeier, 2017). However, since we use non-verbal, non-symbolic addition problems that minimize engagement of linguistic and memory retrieval systems and demand numerical estimation and computation, our predictions regarding the expected representations and associated functional ERP response patterns are somewhat different from the those of Dickson and Federmeier (2017).

Previous work has shown that behavioral responses to non-symbolic arithmetic problems are approximate in nature, systematically dependent on the numerical ratio between the probe and the actual sum (Barth et al., 2008; Barth et al., 2005, 2006; Gilmore et al., 2007, 2010; McCrink et al., 2007). Distinctions between numerical magnitudes that are close to the correct answer are known to be more difficult than those that are distant from the correct answer (Barth et al., 2005, 2006; 2008; Gilmore et al., 2010; McCrink et al., 2007). Thus, there is less likely to be a clear categorical distinction in the mind between the correct answer and incorrect answers that are numerically close to the correct answer in our paradigm as there might be in more highly practiced symbolic arithmetic problems such as those used in single digit symbolic addition or multiplication (e.g., Dickson and Federmeier, 2017). Therefore, like the previous ERP study of arithmetic processing, we will assess sensitivity to correctness in arithmetic verification (i.e., Dickson and Federmeier, 2017). However, we will do so by comparing the brain and behavioral responses to the correct answer with those to incorrect answers that are numerically distant from the correct answer (Correct vs. Incorrect Far) rather than comparing correct outcomes to the average of all incorrect outcomes.

Following Dickson and Federmeier (2017), we will also analyze effects of numerical relatedness during arithmetic verification. Based on the previous behavioral work using non-symbolic arithmetic problems (e.g., Barth et al., 2005, 2006; 2008), we predict that behavioral and brain responses to incorrect answers will be modulated by their numerical relationship to the correct answer. In contrast to how semantic relatedness was defined for multiplication in the previous study using table relatedness (Dickson and Federmeier, 2017), we will assess sensitivity to semantic relatedness, or numerical meaning processing, by comparing responses to incorrect answers that are numerically distant from the correct answer with those incorrect answers that are numerically close to the correct answer (Incorrect Close vs. Incorrect Far). If both hemispheres contribute equally to calculations of correctness and/or semantic relatedness in this form of non-symbolic arithmetic, then there will be main effects of correctness and/or relatedness, but no difference in the brain or behavioral responses by visual field of presentation (i.e., no interaction with visual field of presentation). However, if there are hemispheric asymmetries in arithmetic processing of either type, then we should see differential effects by visual field of presentation (i.e., an interaction between relatedness or correctness and visual field). Previous work on categorization of language and arithmetic suggests that the categorization of correctness should not show hemispheric differences (Dickson and Federmeier, 2017). Similarly, the triple-code model of numerical processing, at least in its original form as we understand it, predicts sensitivity to numerical relatedness in arithmetic should be evident under both left and right visual field of presentation. Other work developmental and neuroimaging work, including a recent ERP study of multiplication, however, suggests that the right hemisphere may be advantaged for processing numerical and arithmetic meaning. If this is the case, sensitivity to numerical relatedness should be evident under left visual field presentation, but either to a lesser extent or not at all under right visual field presentation.

Finally, previous work has also shown systematic overestimation addition problems (and underestimation for subtraction problems), an effect termed Operational Momentum (OM) effect (McCrink et al., 2007; McCrink and Wynn, 2009; Knops et al., 2014). If the form of arithmetic we administered involves OM, then participants are likely to overestimate the sum in our addition problems as well. As a result, trials where the probe is greater than the sum are likely to be more difficult than trials where the probe is less than the sum (i.e., the directional relationship of the probe to the actual sum). As a secondary, completely exploratory factor, then, our analysis considered whether incorrect trial probes/outcomes were greater than or less than the actual sum to determine whether OM effects were present and/or interacted with numerical relatedness effects.

1. Methods

1.1. Participants

Based on pilot data with the same stimuli and paradigm, a sample size of at least 31 participants was determined to be needed to detect the weakest effect of interest in the primary analysis, an interaction between visual field of presentation and relatedness (effect size η^2 =.204) with 95% power. A total of 40 participants were run in anticipation of some of those having to be eliminated based on artifacts. Thirty-four participants (mean age = 24.026 years, SD = 3.555 years) were included in the final dataset. Data from six additional participants were eliminated from the analysis because they retained less than 33% of total trials seen in at least one of the conditions after automatic artifact detection and trial rejection (n = 5) ,[1] or for responding before the instructed response period on a significant number of the trials (over 10% of the total trials, n = 1). All participants were right-handed and had normal or corrected to normal vision. All participants were recruited through a psychology study pool and paid $10 per hour for the participation. The study was conducted under the approval and supervision of the Office for the Protection of Human Subjects at the University of Illinois.

1.2. Procedure

Participants were given a non-symbolic addition task (Barth et al., 2005, 2006; 2008; McCrink et al., 2007). The task began with a green fixation point (diamond) presented at the center of the screen for 500 ms (see Fig. 1). Next, the fixation point remained on the screen as two dot arrays were presented sequentially for 200 ms each at the center of the screen with an inter-stimulus interval of 700 ms. The brief presentation time for the dot arrays was consistent with previous studies as sufficient to estimate numerosity while discouraging exact counting or excessive eye movements (Cicchini et al., 2016; Hyde and Spelke, 2009; Park et al., 2015). Participants were verbally instructed to add the two arrays together in their mind to formulate an estimate of the sum. The fixation point then turned red for 1500 ms, reminding participants to maintain

[1] One subject's EEG data only contained the first 361 trials (out of the 432 total trials) due to the malfunction of the recording system (while the subject's all behavioral data was saved). Since the subject had enough number of artifact free trials per each condition (24 out of 72 by each Visual Field by Distance/Correctness), we included the subject' data into further ERP data analyses.

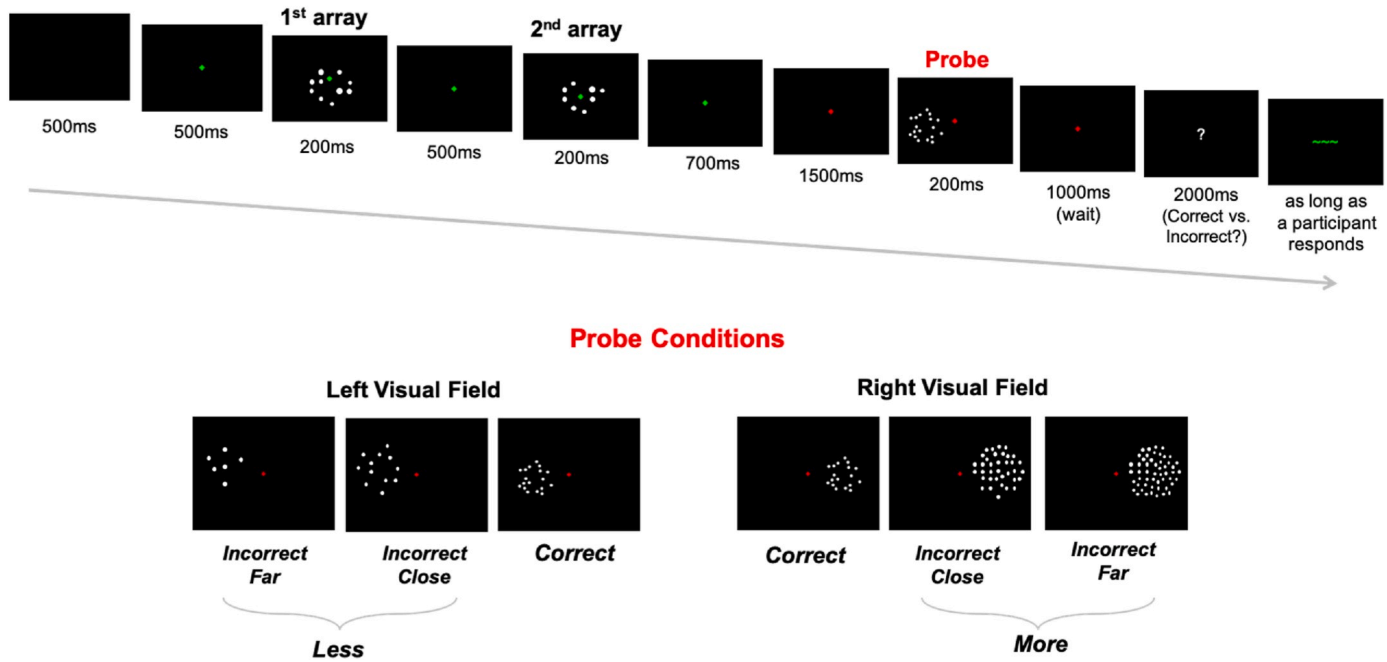

Fig. 1. A schematic of the non-symbolic addition task. Participants were asked to verify whether a probe dot array appearing to the left or right of fixation was the correct or incorrect sum of two dot arrays presented sequentially beforehand. Three factors were manipulated: visual field of presentation (left or right), semantic-numerical relatedness (Correct, Incorrect Close, Incorrect Far) of the probe to the sum, and direction for incorrect trials (i.e, if the probe was More or Less than the sum).

fixation, avoid eye blinks, and to get ready to evaluate the probe array (see Fig. 1). The probe array was then presented to either the left or right visual hemifield for 200 ms. Probe arrays were presented between 4.69° and 11.94° of visual angle to the left or right of the center of the screen to manipulate the hemisphere first receiving stimulation (see Banich, 2003). The probe array was followed by a 1000 ms reflection period where only the red fixation point remained, and participants were told to decide in their mind whether the probe array was correct (the actual sum) or not (see Fig. 1). A question mark then appeared at the center of the screen, prompting participants to press one of two buttons to indicate their answer. The prolonged reflection period and delayed response period were instantiated to avoid motor interference with the ERP components of interest, as in previous studies (Dickson and Federmeier, 2017; Jasinski and Coch, 2012). The participants were told that after making a response they could reset their eyes, blink at will, and then press any button when they were ready to start the next trial. All instructions were given verbally to participants before the study began.

Stimuli were presented on a computer screen that was 110 cm away from a chair where participants sat using E-prime 2.0 software (PST, Pittsburg, PA). As stimuli were presented, the ongoing EEG was recorded using a 128-channel Hydrocel Geodesic Sensor Net and associated NetAmps 300 amplifier (Electrical Geodesics, Inc.). Net Station 4.5 was used to record the EEG signal on a Macintosh desktop computer. Responses were collected via a proprietary button box associated with the E-prime software.

1.3. Design

To investigate the brain and behavioral correlates of non-symbolic addition, we systematically manipulated the semantic-numerical relationship, or relatedness, between the probe array and the sum (of the first two arrays). Relatedness of the probe to the sum was defined as the numerical ratio between the number of dots in the probe array to the actual sum (of the first two arrays (Barth et al., 2005, 2006; 2008; McCrink et al., 2007; McCrink and Wynn, 2009). Probes of three numerical distances were presented to participants: an array with the actual sum (Correct, 1:1 ratio), an incorrect but numerically close array (Incorrect Close, 1:2 ratio), an incorrect and numerically distant array (Incorrect Far, 1:3 ratio). To better understand the contributions of each hemispheric to arithmetic, probes were presented to the right or left hemisphere using the visual half-field design (Left Visual Field, LVF; Right Visual Field, RVF). There were 72 unique arithmetic trials presented to each of the 2 visual fields repeated in 3 blocks for a total of 432 total possible trials (see Table S1). Equal numbers of trials were presented for each of the three 3 distances. In addition, the first addend was larger than the second in half the trials and half of the trials presented probes that were less than the sum (and the other half presented probes that were more than the sum). Response buttons were kept consistent throughout the experiment for each participant, (e.g., "incorrect" response associated with the left-most side of the button box and the "correct" response associated with the button on the right-most side of the button box), but counterbalanced between participants.

1.4. Stimuli

Stimulus arrays were 14 cm by 14 cm images of dots. Addends ranged from 8 to 46 dots and probes ranged from 18 to 78 dots (Table S1). Images were created using a custom MATLAB scripts (see Dehaene et al., 2005; Piazza et al., 2004). Non-numerical magnitudes in the images varied between images, as to not be predicative of number over the course of the experiment, an established method for controlling for non-numerical visual parameters in experiments involving dot stimuli conveying numerical information (see Dehaene et al., 2005; Piazza et al., 2004). Specifically, half of all the dot images were equated on intensive parameters (individual item size and mean inter-item space) and the other half varied but were equated, on average, on the extensive parameters (mean total surface area) (see Hyde and Spelke, 2009, 2011; Hyde et al., 2014; Hyde and Wood, 2011; Khanum et al., 2016; Piazza et al., 2004, 2007 for similar controls). For each of the three dot arrays presented on every trial for every participant, the type of non-numerical control (intensive or extensive) was randomly selected such that there was no reliable relationship between the non-numerical aspects of the arrays presented and the answer on a given trial or across the experiment as a whole. This randomization procedure was done uniquely for every subject, such that non-numerical property combinations were also unique for every participant. These controls eliminate the possibility that non-numerical cues could be used systematically to obtain an answer to our problems for a single subject as well as some systematic bias in the relationship between the answer across all subjects in the final dataset (see Dehaene et al., 2005 Hyde et al., 2014; Khanum et al., 2016; Piazza et al., 2004, 2007). The addend ranges were similar between the distance conditions (8–46 for Correct; 8–45 for Incorrect Far; 8–46 for Incorrect Close; see Appendix for details). The ranges of the sums and probes were equated (18–78 dots; Table S1). Over the entire stimulus set, the number of dots in first or second addends was not related to the probe answer ($r = -0.24$, $p > 0.1$) or the ratio between the actual sum and probe ($r =$ -.05, $p > 0.7$).

1.5. Data acquisition and processing

Standard data acquisition and processing parameters were used to be in-line with our previously published work in numerical cognition using an EGI 128 channel EEG system with adults (e.g., Hyde and Spelke, 2009; Hyde and Wood, 2011). Specifically, the ongoing EEG was recorded at 250 Hz, digitally filtered online between 0.1 and 100 Hz, and referenced online to the vertex. Offline, data was bandpass filtered between 0.1 and 30Hz and segmented into epochs from 100 ms before to 1000 ms after the onset of the probe image. Automatic artifact detection tools were used to detect and reject segments containing eye blinks (max/min difference 140 μV in vertical eye channels), eye movements (max/min difference > 55 μV in horizontal eye channels), and bad channels (max/min difference > 200 μV over entire segment). Channels identified as bad in over 20% of the segments were marked bad over the entire recording. Segments with eye blinks or eye movements were removed from further analysis. Bad channels in remaining segments were replaced using spherical spline interpolation. Artifact-free segments in each experimental condition were averaged, re-referenced to an average reference, and baseline corrected from the average amplitude from −100 ms to probe onset.

1.6. Statistical analyses

To test the effects of arithmetic verification on behavior and the brain response we conducted two focused analyses on the response to the probe array: an analysis of the effects of Correctness (Correct vs. Incorrect Far) and an analysis of numerical Relatedness (Incorrect Far vs. Incorrect Close) of the probe to the sum (see rationale above).

Behavioral. First, we conducted one-sample t-tests to determine whether accuracy of each condition (i.e., Incorrect Far, Incorrect Close, and Correct) was above the chance level (chance = 50%). For the analysis of Relatedness on accuracy, we employed a 2 × 2 × 2 repeated measures analyses of variance (ANOVA) with two levels of Visual Field (LVF or RVF), Probe Direction (Less or More), and semantic Relatedness (Incorrect Far and Incorrect Close). For the analysis of Correctness on accuracy, we conducted a 2 × 2 repeated measures ANOVA with the factors of Visual Field (LVF or RVF) and Correctness (Incorrect Far and Correct). We did not analyze reaction times given that our procedure required participants to delay their responses for 1000 ms after the onset of the probe in order to eliminate motor activity interference with the ERPs to the probe.

ERP. Our ERP analysis targeted two well established components of interest identified as relevant in previous work: the P3 and the Late Positive Complex (LPC). To temporally define these components, we drew directly from a recent ERP study of arithmetic processing using a half visual field presentation paradigm (Dickson and Federmeier, 2017). For the P3, we used the same time window as Dickson and Federmeier (2017), 250-400 ms. For the early part of the LPC, we also used the same time window as Dickson and Federmeier (2017), 400-600 ms. However, LPC is a large and slow-going waveform and, in our paradigm, extended well beyond 600 ms. Given its prolonged nature and complex morphology, we extracted the mean amplitude over two additional uniform time windows matched in length to those of the earlier time window transferred from the Dickson and Federmeier (2017) parameters, (600-800 ms & 800-1000 ms) to characterize the remainder of the LPC. Thus, LPC was characterized by three separate time windows of interest (400-600 ms, 600-800 ms, or 800-1000 ms) corresponding to early, middle, and late portions of the slow going positivity. Broad time windows of 200 ms conservatively estimated the average effects over many time samples (50 time points) and, thus, errored of the side of caution to avoid spurious differences between conditions more likely to emerge over individual time points or more narrow time windows (see Luck, 2014; Luck and Gaspelin, 2017).

Since the previous study of hemispheric differences used a low-density 26 channel EEG system (Dickson and Federmeier, 2017), we were not able to directly translate electrode sites to our high-density 128 channel EEG system. Instead, we employed a data-driven approach to channel selection for each participant, constrained by established topography of the P3 and LPC components. Specifically, we used functional data to independently define channels of interest (COIs) over central and posterior parietal sites for each participant. This method is adopted from standard approaches to data reduction in functional magnetic resonance imaging (Dufour et al., 2013; Saxe et al., 2006). We employed a split-half method to avoid non-independence errors (Vul et al., 2009). To do this, we divided data into odd trials and even trials. Next, we contrasted the condition of interest for the Relatedness (i.e., Incorrect Far minus Incorrect Close) and Correctness (Incorrect Far and Correct) analyses in each channel over 67 central (EGI Hydrocel 128 Channels: 6, 7, 13, 29, 30, 31, 35, 36, 37, 41, 42, 47, 55, Cz, 80, 87, 93, 98, 103, 104, 105, 106, 110, 111, 112), left (50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74), and right (76, 77, 78, 79, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 94, 95, 96, 97, 99, 100, 101) posterior sites for each time window of interest (defined above) for odd and even trials separately. For each participant, we then identified the 6 electrodes for each of the 3 scalp regions (6 central, 6 left posterior, and 6 right posterior channels) with the greatest (positive) difference value on the average of odd trials and used those electrodes as the channels of interest from which to extract data from the even trials, thus defining channels of interest on data independent that used for analysis (see Vul et al., 2009). We then did the reverse for each participant, extracting data from the even trials based on the 18 channels of interest defined over the odd trials. Finally, we averaged amplitudes extracted from the independent COIs for the odd and even trials for each time window for each participant. This approach uses independent data to identify the channels most sensitive to the functional signatures of interest while avoiding errors of non-independence (Vul et al., 2009). To analyze Correctness, the P3 mean amplitude was then input into 2 × 2 repeated measure of ANOVA with the factors of Visual Field (LVF and RVF) and Correctness (Incorrect Far and Incorrect Close). To analyze Relatedness, LPC mean amplitude data was inputted into a 2 × 2 × 2 repeated measure of ANOVA with the factors of Visual Field (LVF and RVF), Direction (Less (Probe < Sum) and More (Probe > Sum)), and Relatedness (Incorrect Far and Incorrect Close).

Reliability statistics for all the main analyses can be found in Table S2 of the supplementary materials.

2. Results

2.1. Accuracy

Performance relative to chance. Participants were above chance (50%) in their accuracy on the Correct (M = 0.619, SD = 0.139, t (33) = 5.000, $p < 0.001$, $d = 0.857$) and Incorrect Far conditions (M = 0.884, SD = 0.076, t (33) = 29.815, $p < 0.001$, $d = 5.113$), but not on the Incorrect Close condition (M = 0.522, SD = 0.122, t (33) = 1.041, $p =$

0.306, $d = 0.178$).

Correctness. There was a main effect of Correctness on accuracy (F (1, 33) = 77.138, $p < 0.001$, $\eta^2 = 0.700$; Table 1; Fig. 2), with accuracy for identifying incorrect (far) probes being greater than that of correct probes. There was no main effect of Visual Field (F (1, 33) = 1.775, p = 0.192, $\eta^2 = 0.051$), but there was an interaction between Correctness and Visual Field (F (1, 33) = 5.065, $p = 0.031$, $\eta^2 = 0.133$; Table 1; Fig. 2). Post-hoc analyses revealed that accuracy for correct probes was greater under RVF presentation compared to LVF presentation (RVF (M = 0.625, SD = 0.140) > LVF (M = 0.596, SD = 0.140); F (1, 33) = 5.573, $p = 0.024$, $\eta^2 = 0.144$), but there were no difference in accuracy between the visual fields of presentation when the probe was incorrect and distant from the correct answer (F (1, 33) = 0.545, $p = 0.466$, η^2 = 0.016).

Relatedness. There were main effects of Relatedness (F (1,33) = 340.856, $p < 0.001$, $\eta^2 = 0.912$; Table 2), Direction (F (1,33) = 23.649, $p < 0.001$, $\eta^2 = 0.417$; Table 2), and an interaction between Relatedness and Direction (F (1,33) = 20.237, $p < 0.001$, $\eta^2 = 0.380$; Table 2) on accuracy. On average, incorrect probes that were numerically distant from the actual sum were identified more accurately compared to incorrect probes that were numerically closer to the actual sum, displaying a classic numerical distance effect as is routinely observed in tasks involving numerical comparison (Barth et al., 2005, 2006; Temple and Posner, 1998; Halberda and Feigenson, 2008; Szűcs and Csépe, 2005). Furthermore, incorrect probes that were less than the actual sum were identified more accurately (M = 0.780, SD = 0.123) than incorrect probes that were more than the sum (M = 0.645, SD = 0.119), showing a classic operational momentum effect often observed for arithmetic (e.g., McCrink et al., 2007). The interaction seemed to be driven by the fact that the effect of Relatedness, or difference between accuracy on incorrect close versus incorrect far trials, was greater in problems where the probe was more numerous than the actual sum (F (1,33) = 123.437, $p < 0.001$, $\eta^2 = 0.789$; Table 2) compared to problems where the probe was less numerous than the actual sum (F (1,33) = 429.444, $p < 0.001$, $\eta^2 = 0.929$; Table 2). There was also an interaction between Visual Field of presentation and Direction (F (1,33) = 6.826, $p = 0.013$, $\eta^2 = 0.171$; Table 2). The interaction appeared to be driven by the fact that the Direction effect, or operational momentum, was more pronounced when the probe was presented to the LVF (LVF: F (1,33) = 33.201, $p < 0.001$, $\eta^2 = 0.502$; RVF: F (1,33) = 22.708, $p < 0.001$, $\eta^2 = 0.408$; Table 2; Fig. 3) (Ansari, 2010).

2.2. Event-related potentials

We analyzed the effects of Visual Field of stimulus presentation by Correctness and Relatedness on ERPs during four distinct time windows. It may be important to reiterate that we averaged the response across left and right scalp sites and do not analyze scalp site as a factor, as electrophysiology from either hemisphere propagates across the entire scalp and previously reported results find no interactions between relatedness and hemisphere of electrode group (Dickson and Federmeier, 2017).

Correctness. There were no main effects or interactions with Correctness on P3 amplitudes ($ps > 0.2$; Table 3) or the early LPC ($p > 0.1$). However, main effects of Correctness emerged on the middle and late portions of the LPC (Incorrect Far > Correct, $ps < 0.05$; Table 3; Fig. 4), where mean amplitudes were greater in response to Incorrect Far probes compared to Correct probes. There were no additional main effects or interactions with Correctness ($ps > 0.6$; Table 3).

Relatedness. There were no main effects of or interaction with Relatedness on the P3 ($ps > .05$; Table 4). Main effects of Relatedness and interactions between Relatedness and Visual Field of presentation emerged in the early and middle portions of the LPC. Further analyses revealed that Relatedness effects were sustained during both time windows when probes were presented to the LVF (early: F (1,33) = 10.476, $p = 0.003$, $\eta^2 = 0.241$; middle: F (1,33) = 12.100, $p < 0.001$, $\eta^2 = 0.268$; Table 4; Figs. 4 and 5) but not when presented to the RVF (early: F (1,33) = 0.010, $p = 0.921$, $\eta^2 < 0.001$; middle: F (1,33) = 1.015, $p = 0.321$, η^2 = 0.030; Table 4; Figs. 4 and 5). These effects did not persist into the latest

Table 1
Effects of correctness on accuracy.

Factor(s)	F	p	Partial η^2
Visual Field	1.775	.192	.051
Correctness	77.138	<.001**	.700
Visual Field × Correctness	5.065	.031*	.133
Visual Field/Incorrect Far	.545	.466	.016
Visual Field/Correct	5.573	.024*	.144

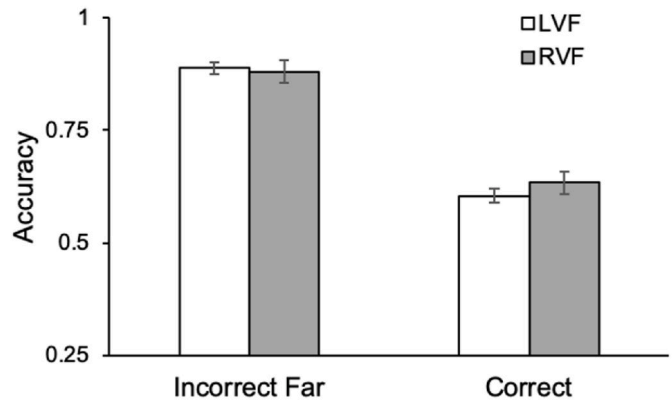

Fig. 2. Behavioral results for the analysis of correctness. The error bars represent the -/+ 1 standard error.

Table 2
Effects of numerical relatedness on accuracy.

Factor(s)	F	p	Partial η^2
Visual Field	.327	.571	.010
Relatedness	340.856	<.001**	.912
Direction	23.649	<.001**	.417
Visual Field × Relatedness	.051	.823	.002
Visual Field × Direction	6.826	.013*	.171
Direction/LVF	33.201	<.001**	.502
Direction/RVF	22.708	<.001**	.408
Relatedness × Direction	20.237	<.001**	.380
Relatedness/Less	123.437	<.001**	.789
Relatedness/More	429.444	<.001**	.929
Visual Field × Relatedness × Direction	1.989	.168	.057

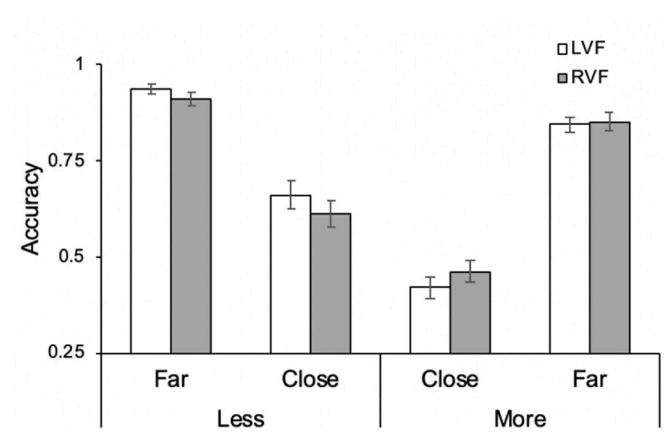

Fig. 3. Effects of distance and direction on accuracy. The error bars represent the -/+ 1 standard error.

Table 3
Effects of correctness on event-related potentials.

Component	Factor(s)	F	p	Partial η^2
P3 (250-400 ms)	Correctness	.336	.566	.010
	Visual Field	.007	.932	<.001
	Correctness × Visual Field	1.666	.206	.048
Early LPC (400-600 ms)	Correctness	2.224	.145	.063
	Visual Field	3.494	.070	.096
	Correctness × Visual Field	.190	.666	.006
Middle LPC (600-800 ms)	Correctness	4.660	.038*	.124
	Visual Field	.290	.594	.009
	Correctness × Visual Field	.093	.762	.003
Late LPC (800-1000 ms)	Correctness	4.525	.041*	.121
	Visual Field	.397	.533	.012
	Correctness × Visual Field	.081	.778	.002

portion of the LPC. However, there was a main effect of Direction during the late LPC, (F (1,33) = 4.391, $p = 0.044$, $\eta^2 = 0.117$; Table 4; Fig. 6), with amplitudes being greater when the probe array was less than the actual sum (M = 0.827, SD = 0.412) compared to when the probe array was more than the actual sum (M = 0.530, SD = 0.412). It may be important to reiterate that we averaged the response across left and right scalp sites and do not analyze scalp site, as electrophysiology from either hemisphere propagates across the entire scalp and previously reported results find no interactions between relatedness and hemisphere of electrode group (Dickson and Federmeier, 2017).

3. Discussion

The main purpose of the current study is to investigate hemispheric contributions to processing numerical meaning in arithmetic. We tested the theory that both hemispheres contribute equally to arithmetic processing against the idea that arithmetic processing may show an asymmetry favoring the right hemisphere. To do this, we measured event-related brain potentials as participants completed a non-verbal, non-symbolic arithmetic verification task using a visual half-field presentation paradigm (e.g., Banich, 2003; Dickson and Federmeier, 2017). The visual half-field paradigm allowed us to experimentally manipulate which hemisphere received arithmetic outcomes first, thereby providing a hemispheric advantage to the hemisphere contralateral to the visual field of presentation. Our non-symbolic, non-verbal numerical stimuli (dot arrays) were used in an attempt to isolate processing of numerical meaning in arithmetic from other associated general memory and linguistic processes (Barth et al., 2005, 2006; 2008; McCrink et al., 2007). Specifically, these arithmetic problems required sums to be approximated online and, as such, could not be readily solved by retrieving an answer from memory. The use of dot arrays that varied in item size, spacing, and item position, and that participants had never seen reduced automatic associations with memorized numerical symbols and language. With these stimuli and paradigm, we analyzed behavioral and brain sensitivity to three aspects of numerical meaning: sensitivity to whether probes were the correct sum or not, sensitivity to the numerical relationship of the incorrect probes to the actual sum, and sensitivity to whether incorrect probes were greater or less than the actual sum.

3.1. Processing of numerical meaning in arithmetic

We found that several aspects of the brain and behavioral response to probe arrays were dependent on its numerical relationship to the actual sum of the two addends. Specifically, early and middle portions (400-600 ms, 600-800 ms) of the LPC showed a numerical relatedness effect, whereby ERP amplitudes to the probe array scaled with their numerical ratio relationship to the sum. The sum was never shown and had no visual properties of its own; visual properties and non-numerical magnitudes of the addends were always different from those of the probe. The fact that responses to the probe were sensitive to the numerical relationship to the sum despite the sum never being shown indicates that participants were performing mental arithmetic to some degree of accuracy and the brain was contrasting the numerical value of the mentally computed sum to that of the probe array. In addition, we saw a classic distance effect in behavioral accuracy, where accuracy for probes that were numerical distant from the actual sum was higher than accuracy for incorrect probes that were closer to the correct answer (Barth et al., 2005, 2006; 2008; McCrink et al.., 2007; Pica et al., 2004). There was also substantial confusion between the probes that were close to the actual sum and probes that were the correct sum. In the case of probes that were incorrect but numerically close, accuracy was actually below chance. This suggests that participants were, in fact, representing the approximate rather than an exact, memorized sum of the addends.

3.2. Processing of correctness in arithmetic

Given the approximate nature of the mental representations, it is not surprising that we did not observe effects of correctness on the P3, a component often thought to characterize explicit recognition and categorization in the decision making process (Comerchero and Polich,

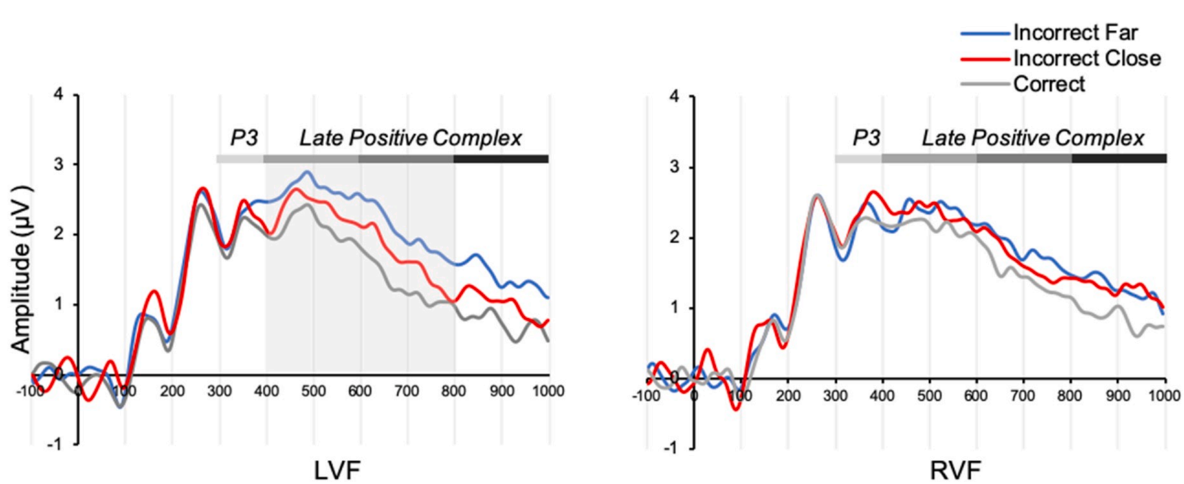

Fig. 4. Average event-related potentials over parietal electrode sites for each type of probe under left and right visual field presentation. The portion of the waveform shaded by gray indicates the time window(s) of statistical significance ($p < 0.05$) between the Incorrect Far and Incorrect Close conditions.

Table 4
Effects of numerical relatedness on event-related potentials.

Component	Factor(s)	F	p	Partial η^2
P3 (250-400 ms)	Relatedness	.198	.660	.006
	Direction	.025	.875	.001
	Visual Field	.202	.656	.006
	Relatedness × Direction	.031	.861	.001
	Relatedness × Visual Field	3.586	.067	.098
	Direction × Visual Field	2.449	.127	.069
	Relatedness × Direction × Visual Field	.744	.395	.022
Early LPC (400-600 ms)	Relatedness	6.312	.017*	.161
	Direction	1.522	.226	.044
	Visual Field	.004	.948	<.001
	Relatedness × Direction	.275	.604	.008
	Relatedness × Visual Field	5.306	.028*	.139
	Relatedness/LVF	10.476	.003**	.241
	Relatedness/RVF	.010	.921	<.001
	Direction × Visual Field	.253	.618	.008
	Relatedness × Direction × Visual Field	1.096	.303	.032
Middle LPC (600-800 ms)	Relatedness	10.593	.003**	.243
	Direction	1.408	.244	.041
	Visual Field	.732	.398	.022
	Relatedness × Direction	.133	.718	.004
	Relatedness × Visual Field	4.627	.039*	.123
	Relatedness/LVF	12.100	.001**	.268
	Relatedness/RVF	1.015	.321	.030
	Direction × Visual Field	.633	.432	.019
	Relatedness × Direction × Visual Field	1.109	.300	.033
Late LPC (800-1000 ms)	Relatedness	2.418	.129	.068
	Direction	4.391	.044*	.117
	Visual Field	.800	.377	.024
	Relatedness × Direction	.053	.820	.002
	Relatedness × Visual Field	3.230	.081	.089
	Direction × Visual Field	1.511	.228	.044
	Relatedness × Direction × Visual Field	.940	.339	.028

1999; Donchin and Coles, 1988; Jasinski and Coch, 2012; Luck, 2014). The lack of P3 effects contrasts with findings from the previous work on arithmetic (Dickson and Federmeier, 2017; Jasinski and Coch, 2012; Niegedden and Rösler, 1999). However, in those cases, stimuli involved symbolic numbers and arithmetic operations that are known to more heavily involve the retrieval of exact answers from memory (Dickson and Federmeier, 2017; Jasinski and Coch, 2012; Niegedden and Rösler, 1999). Given participants were uncertain about whether incorrect close probes and, to some degree, correct probes were actually correct or not, it may be the case that there was not a consistent recognition response to differentiate the probe outcomes. This could either be because participants never really recognized the correct response as they might in exact, symbolic addition problems or because the probe could not be evaluated quickly enough, resulting in a less consistent timing and subsequent blurring of the response across later portions of the ERP over averaging.

Somewhat marginal effects of correctness where, however, seen later in the ERP processing stream. Specifically, middle and late portions of the LPC were more positive for the Incorrect Far compared to Correct probes. Behavioral accuracy was robustly sensitive to correctness, with participants being significantly more accurate to identify incorrect probes that were distinct from the correct answer compared to correct probes. The emergence of a marginal brain sensitivity to correctness happening after brain sensitivity to numerical relatedness, combined with a robust effect of correctness on accuracy from a behavioral response collected after the ERP epoch ended suggests that numerical meaning was computed before correctness in our task. This makes sense given that answers were approximated, and participants were asked to withhold execution of their decision until after the ERP epoch ended. This temporal order of numerical relatedness and correctness judgement effects differed from that observed in a recent study of symbolic multiplication using a similar visual half-field paradigm (Dickson and Federmeier, 2017). Given the effects of correctness were marginal, they should be replicated. Should they replicate, future work may probe whether this temporal relationship between processing of numerical meaning and categorization of correctness is specific to the context of non-symbolic arithmetic verification under uncertainty or generalizes to

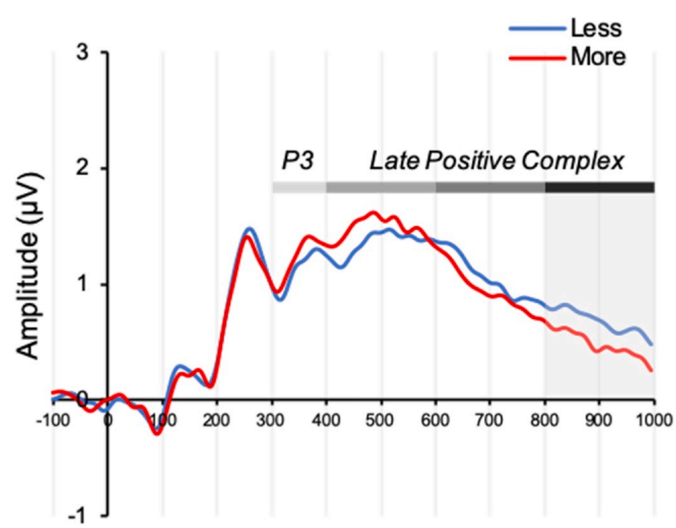

Fig. 6. Effects of direction on late time window of the LPC (800-1000 ms). The portion of the waveform shaded by gray indicates the time window(s) of statistical significance ($p < 0.05$) between conditions.

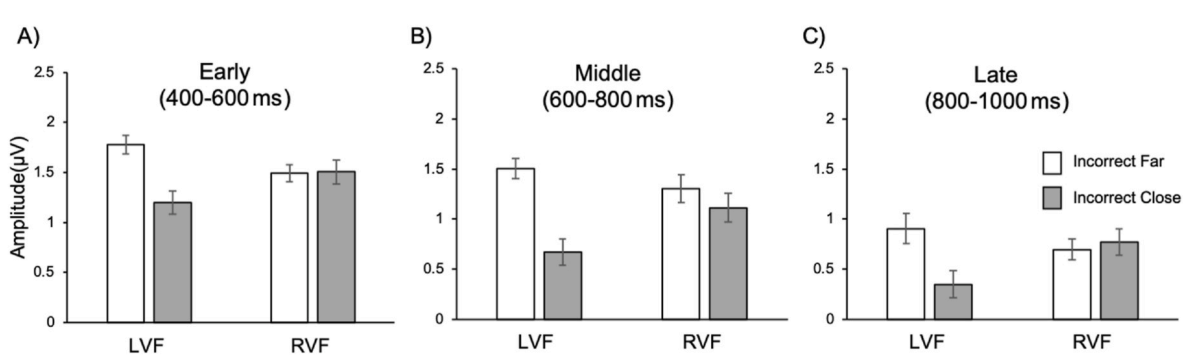

Fig. 5. Average LPC response to incorrect probes by visual field of presentation. A). Early LPC B). Middle LPC C). Late LPC. Error bars represent the -/+ 1 standard error.

other addition contexts more broadly.

3.3. Hemispheric asymmetries

We found at least two indices of hemispheric asymmetries in arithmetic. First, we found clear evidence of a right hemispheric asymmetry in the processing of numerical meaning in the ERP response. Specifically, the amplitudes of early and middle portions of the LPC (400-800 ms) to incorrect probes were systematically modulated by the numerical relatedness to the actual sum under left visual field presentation, but not right visual field presentation. These results seem to indicate that regions in the right hemisphere, but not the left represent the numerical magnitude relationship between the actual sum and the probe in our experimental context. More broadly, these results are consistent with another recent study of symbolic multiplication using a visual half-field paradigm, also showing that the LPC was sensitive to numerical relatedness under left, but not right visual field presentation (Dickson and Federmeier, 2017). In the case of this previous study, right lateralization could have resulted from numerical-semantic associations with the multiplication product being retrieved from memory. Our results complement these findings by also showing a right-lateralization bias is processing numerical meaning under conditions that do not allow retrieval of an exact answer and where numerical meaning has to be computed within the experimental trial itself. Together the results of our study and the previous study of symbolic multiplication suggest that representing numerical meaning in arithmetic may be biased towards the right hemisphere.

Developmental research also supports the idea that the cortical foundations for thinking about numerical magnitudes and arithmetic operations are in the right hemisphere. Processing of non-symbolic numerical magnitudes is initially right lateralized in infancy and becomes bilateral over child development (Ansari and Dhital, 2006; Cantlon et al., 2006; Edwards et al., 2016; Emerson and Cantlon, 2015; Hyde et al., 2010; Izard et al., 2008; Libertus et al., 2009). By adulthood, both symbolic and non-symbolic numerical processing engages bilateral regions of the parietal lobe (Ansari and Dhital, 2006; Cantlon et al., 2006; Emerson and Cantlon, 2015; Rivera et al., 2005; Rosenberg-Lee et al., 2011; Piazza et al., 2004, 2007). Nevertheless, even in educated adults, tasks that engage more basic numerical magnitude processing and computation often more strongly engage right compared to left parietal regions (Holloway and Ansari, 2010; Holloway et al., 2010; Chassy and Grodd, 2012; Cohen Kadosh et al., 2007). The working hypothesis, then, is that numerical magnitude representation and manipulation is founded in right-lateralized regions of the brain. These findings may suggest an amendment to the triple-code model of numerical processing (Dehaene et al., 2003), which posits that numerical meaning is processed in bilateral parietal regions.

Second, we found that the visual field of presentation influenced behavioral accuracy for correctness judgements, where accuracy was greater for correct probes under RVF presentation compared to LVF presentation. This might suggest a more prominent role of the left hemisphere in integrating information for categorization or decision making related to arithmetic (Funnell et al., 2007; Hellige, 2001; Hellige and Michimata, 1989; Kosslyn et al., 1989). Interestingly, we did not find a differential effect of visual field of presentation on sensitivity to correctness in the brain response. That is, ERP amplitudes to probes that correctly matched the actual sum were significantly smaller than that those to incorrect and distant probes between 600 and 1000 ms, but this effect was not dependent of visual field of presentation (i.e., no interaction between visual field of presentation and correctness). A hemispheric asymmetry for correctness judgements in the behavioral response but not in the brain response may be explained by the fact that participants were instructed to delay the behavioral categorization (correct or incorrect) until after the ERP epoch of interest was over. If hemispheric asymmetries in correctness judgements result from the execution of the decision-making processing itself, then our ERP epoch would not have been sensitive to this process and, in our study, behavioral responses would be the only measure of those processes. Future work could follow up on this, possibly through eliminating the delay, designing a task that also includes recording during the decision process, and time locking the decision response.

3.4. Operational Momentum

Although only an exploratory aspect of our investigation, we observed an effect of whether the probe was greater or less than the actual sum on behavior and the brain response. Specially, amplitudes during the late portion of the LPC were larger when probes were less than the actual sum compared to when probes were greater than the actual sum. Accuracy was also greater for deciding that the probe was less than the actual sum, equating for the ratio of comparison. These findings are consistent with previous behavioral work on operational momentum (OM) in arithmetic, where addition characteristically leads to an overestimation of the answer (and subtraction leads to underestimation) (Knops et al., 2009; McCrink et al., 2007). Subsequently, a probe that is less than the actual sum should be easier to judge as incorrect compared to an incorrect probe that is equidistant from the sum but greater because the actual sum is overestimated, moving the probe and the estimated sum mentally closer.

Our behavioral findings align nicely with the previous behavioral work on operational momentum, while the nature and time course of our ERP findings may add new insight into the nature of this mental arithmetic phenomenon. More specifically, we observe that sensitivity to numerical relatedness, associated with statistical effects of numerical distance, occurs earlier and is temporally distinct from OM effects. The finding that OM effects come later and are distinct from processing of initial numerical meaning is inconsistent with several prominent cognitive accounts of OM, which explain OM as systematic estimation error into the mental representation that arise through an interaction between the direction of the operation (i.e., addition vs. subtraction) and the calculation of numerical meaning (Chen and Verguts, 2012; McCrink et al., 2007; McCrink and Hubbard, 2017; Knops et al., 2014; Pinhas and Fischer, 2008). If these theories are accurate, we would have expected sensitivity to relatedness to interact with direction of the probe (greater or less than sum). However, with the temporal resolution of the ERP we are able to see that sensitivity to direction and numerical relatedness have distinct functional signatures and time courses. These findings may provide support for another theory of OM that posits that OM arises from a strategy or heuristic in decision making process rather than during the calculation of numerical meaning in arithmetic (McCrink et al., 2007; McCrink and Hubbard, 2017). Since this is the first study to our knowledge to use a high temporal resolution measure of arithmetic processing to examine operational momentum, further work will be needed to confirm the suggested interpretations here. Nevertheless, our results suggest ERPs are a promising avenue to test competing theories of OM.

4. Conclusions

While humans employ many cognitive abilities and strategies to successfully and efficiently do mental arithmetic, we attempted to focus on the hemispheric contributions to processing numerical meaning in arithmetic. We did this by using a non-verbal, non-symbolic addition task under visual half-field presentation. This allowed us to study the processing of numerical meaning in arithmetic when answers had to be calculated and could not be retrieved from memory, as well as reduce the influence of automatic associations with language processing. The visual half-field presentation allowed us to manipulate which hemisphere received the information first while ERPs allowed us to track functional brain sensitivities to these factors across time with high resolution. In this context, we provide new evidence of hemispheric asymmetries in arithmetic. Specifically, the processing of numerical meaning in arithmetic appears to be biased towards the right hemisphere, whereas integrating information for categorization or decision making related to the correctness of arithmetic problem may be advantaged in the left hemisphere. Our study also revealed that processing of numerical relatedness in arithmetic occurred before sensitivity to correctness verification. Finally, our exploratory analyses showed that effects of operational momentum in arithmetic may be separate from the mental representation of numerical meaning and arithmetic processing itself. This finding provides novel evidence against accounts that posit operational momentum occurs as part of the computation of numerical meaning in arithmetic and provides novel support for theories that posit momentum to occur as part of the decision-making process. More broadly, these findings call for amendments to current theories of functional brain organization for arithmetic by providing novel evidence of differing functional roles for the left and right hemispheres and providing a novel cognitive account of how functional processes related to arithmetic processing unfold over time.

Author contributions

Selim Jang: Conceptualization, Methodology, Software, Formal analysis, Investigation, Data Curation, Writing-Original Draft. **Daniel C. Hyde:** Conceptualization, Methodology, Writing - review & editing, Supervision.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.neuropsychologia.2020.107524.

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