A Unified Framework for Identifiability Analysis in Bilinear Inverse Problems with Applications to Subspace and Sparsity Models*

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Abstract

Bilinear inverse problems (BIPs), the resolution of two vectors given their image under a bilinear mapping, arise in many applications. Without further constraints, BIPs are usually ill-posed. In practice, properties of natural signals are exploited to solve BIPs. For example, subspace constraints or sparsity constraints are imposed to reduce the search space. These approaches have shown some success in practice. However, there are few results on uniqueness in BIPs. For most BIPs, the fundamental question of under what condition the problem admits a unique solution, is yet to be answered. For example, blind gain and phase calibration (BGPC) is a structured bilinear inverse problem, which arises in many applications, including inverse rendering in computational relighting (albedo estimation with unknown lighting), blind phase and gain calibration in sensor array processing, and multichannel blind deconvolution (MBD). It is interesting to study the uniqueness of such problems.

$$\text{(BIP)}\qquad \begin{aligned} \text{find}\quad & (x,y),\\ \text{s.t.}\quad & \mathcal{F}(x,y)=z,\\ & x\in\Omega_{\mathcal{X}},\ y\in\Omega_{\mathcal{Y}}. \end{aligned}$$

$$\text{(BGPC)}\qquad \begin{aligned} \text{find}\quad & (\lambda,X),\\ \text{s.t.}\quad & \operatorname{diag}(\lambda)AX=Y,\\ & \lambda\in\mathbb{C}^n,\ X\in\Omega_{\mathcal{X}}. \end{aligned}$$

In this paper, we define identifiability of a BIP up to a group of transformations. We derive necessary and sufficient conditions for such identifiability, i.e., the conditions under which the solutions can be uniquely determined up to the transformation group. Applying these results to BGPC, we derive sufficient conditions for unique recovery under several scenarios, including subspace, joint sparsity, and sparsity models. For BGPC with joint sparsity or sparsity constraints, we develop a procedure to compute the relevant transformation groups. We also give necessary conditions in the form of tight lower bounds on sample complexities, and demonstrate the tightness of these bounds by numerical experiments. The results for BGPC not only demonstrate the application of the proposed general framework for identifiability analysis, but are also of interest in their own right.

Index terms— uniqueness, transformation group, equivalence class, ambiguity, blind gain and phase calibration, sensor array processing, inverse rendering, SAR autofocus, multichannel blind deconvolution

References

[1] Y. Li, K. Lee, and Y. Bresler, “A unified framework for identifiability analysis in bilinear inverse problems with applications to subspace and sparsity models,” *arXiv preprint arXiv:1501.06120*, 2015.

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