Modeling the Regional Economy I
Two-sector to Multi-sector Models
Introduction
Theories of Regional Growth (1950’s, 1960’s)

- Focus on the **drivers** of development

  Measured in terms of **growth** of **economic indicators** (Keynesian)

- Possible drivers of growth:
  - Demand
  - Production capacity
  - Endowments (resources / production factors)
  - Savings → investments

**Exogenous engine of growth**
Introduction
Theories of Regional Growth (1950’s, 1960’s)

- Circular flow $\rightarrow$ economic multipliers
- François Quesnay (1758): *Tableau Économique*
- Similar notion in Smith, Ricardo and Marx
- Full formulation in Léon Walras (1874)
The Two-Sector Model
Economic Base Framework
The Two-Sector Model

Introduction

• ‘Oldest’ regional economic model
• Ideas within this model permeate in other regional frameworks
• 1950-1980s: Postwar needs of understanding and forecasting the rapid pace of urban growth
• Economic Base Model: causal mechanism for the process of economic growth

Regional export activity: primary determinant of growth
The Two-Sector Model
Origins

Interested in why cities/regions evolved

Werner Sombart (1916): Two interrelated group of activities in a city dynamics:
- **city forming** (*städtegründer*) activities that bring income from outside the city
- **city filling** (*städtefuller*) activities which income dependents upon demand from city forming activities

Geographers developed the notion into 2 types of activities:

1. *Raison d’être* of a region (**Basic Activities**)
2. Complementary activities (**Non-basic Activities**)

→ **Spatial heterogeneity** described by their **basic / non-basic ratios**
The Two-Sector Model

Origins

**Staple Theory** (Harold Innis)
- Study of different development patterns in the Canadian provinces
- Key: small range of commodities (ag. or resources) generate export demand → shape development

- Patterns:
  - East Coast: cod (decentralized)
  - West Coast: wheat (decentralized)
  - Central: fur (centralized, large firms)

- Heartland vs Hinterland → restriction to growth of residentiary industries due to specialization

**Economics**

- **Staple commodity** (main exported good locally)
- **Export Industries** → **Local Income**
- **Infrastructure** → **Residentiary Industries**

≠ staples → ≠ linkages → ≠ potentials for regional growth
The Two-Sector Model
Origins - Synthesis

<table>
<thead>
<tr>
<th>Geographers</th>
<th>Economists</th>
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<tbody>
<tr>
<td>Basic</td>
<td>Export</td>
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<tr>
<td>Non-basic</td>
<td>Local</td>
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</table>

Instead of **basic / non-basic ratios**, economists used **local / total ratios** that enabled the creation of an economic model.
The Two-Sector Model
First Physical Model at Urban Level

- Hoyt’s Model

\[ \frac{L_n}{L_T} = \alpha \rightarrow \text{a share of total employment} \]

\[ 0 < \alpha < 1 \]

\[ L_T = L_b + L_n \]

\[ \Rightarrow L_T = \frac{1}{1 - \alpha} L_b \]

Total Employment

Employment (base sector) (exogenous)

Employment (non-base sector)

\[ > 1 : \text{the multiplier} \]

i.e. basic employment has the capacity to generate additional employment in the region
The Two-Sector Model
The Ripple Effect

- How does the multiplier works?
  - Stone thrown into a pond...

\[
\frac{1}{1 - \alpha} = 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \cdots
\]

- Since $0 < \alpha < 1$, the series converges.
- Hence, the size of the multiplier inform us about the structure of the economy

- The economic base **multiplier** tries to **summarize** the endogenous behavior of households, government and industries in the region
The Two-Sector Model
Assumptions

• Duality of total economic activity

• Non-basic activities depend on basic activities (export stimulus → base → non-exportable)

• Ceteris paribus conditions: incentives do not vary enough to induce factor migration

• Multiplier is constant to any export shock → change in exports composition cannot be assessed
The Two-Sector Model

Implications

- The multiplier reflects the size of basic activities in the region
- Economic cycles: exogenous demand shocks + depend on the size of the base
- Growth leads to specialization around the base sector
- Regional productive specialization is key (increases demand for local products)
The Two-Sector Model

Major Issues

• Does not imply an equilibrium growth rate (no negative feedbacks)
• No information about regional convergence processes (relative growth)
• Unable to define the determinants of growth (exogenously driven)
• Nonspatial → downward bias on multiplier
• Region is a uniform space vs rest of the economy (no interdependence between regions)
• High level of activities aggregation (two sector)
• Ignores capacity constraints
• Assumes perfect elasticity of supply for inputs
• Residentiary industries have important role in allowing the base to expand
The Two-Sector Model
You might be wondering...

- Does the multiplier vary by region?
- Does the multiplier change over time?
- Will the impacts of a change in manufacturing differ from those in services using this model?
- Can we use this model for regions of all sizes?
- What is the role of industrial concentration?
The Two-Sector Model
Does the multiplier vary by region?

• Yes, but why?
  • Function of the structure (economic composition of industry)
  • Function of the size of the economy
  • Function of the way in which the export and local sectors are linked to each other
The Two-Sector Model
Does the multiplier change over time?

- Yes, but why?
  - Structure of the economy changes – some industries grow more rapidly, others decline or close
  - Size of the region may change
  - Relationship between export and local sectors may change (export buy more/less from local)

- Two-Sector Model: No mechanism to explain the diversification of the economy
The Two-Sector Model
Will the impacts of a change in manufacturing differ from those in services using this model?

• **No, but why?**
  • Because we only have **one multiplier** to represent all industrial sectors
The Two-Sector Model
Can we use this model for regions of all sizes?

- Depends...
  - Works best for smaller regions and small metropolitan regions (<1 million population)
  - Larger regions have more complex structure and the role of export activity may not be as significant
The Two-Sector Model
What is the role of industrial concentration?

- Tension in regional development policy between

- Promotion of a region’s comparative (competitive) advantage
  - Problem – subject to cyclical demand
  - Generates larger multiplier but also greater disruption if this industry declines in importance

- Diversify region’s economy
  - More recession-proof but perhaps less competitive?
  - May generate less impact from expansion but gains are in terms of stability
The Two-Sector Model
Implementation: Identifying Basic Activities

(1) Assignment

- Assume that all resource and manufacturing is export and rest is local
- Problem: export of services (such as insurance or banking) may be as extensive as that for manufacturing
- Crude approximation, difficult to defend, use as the last resort!

(2) Survey

- Mail or telephone local firms
- Request breakdown of their sales – within the region and outside the region
- Aggregate over all sectors to generate export and local employment or sales and calculate $\alpha$ and thus the multiplier
- Problem: firms inundated with surveys, expensive and time consuming, privacy issues
The Two-Sector Model
Implementation: Identifying Basic Activities

(2) Survey – Example

- Fortune Magazine (April 1938)
  - Town of Oskaloosa, IA
  - Census: 3,000 families (origin and destination of income flows)
    - business (source of inputs and destination of outputs)
  - Main export industries in 1937: manufacturing of goods
    - professional services
(3) Non-Survey

- Most popular and widely used
- Method most often used is to apply location quotients
- Compare the % allocation of each sector in the region’s total in comparison with a similar ratio at the national level
The Two-Sector Model
Implementation: Identifying Basic Activities (Non-Survey)

Location Quotients

- Measure the capacity of a particular sector to supply its own regional demand by assessing its concentration in the region in relation to the nation.
- Minimal information required:

\[ LQ_i^r = \left( \frac{x_i^r}{x^r} \right) / \left( \frac{x_i^{nation}}{x^{nation}} \right) \]

- \( LQ < 1 \) : non-basic activity
- \( LQ > 1 \) : basic activity → surplus level of employment/income is assumed to measure regional export capacity
The Two-Sector Model
Implementation: Identifying Basic Activities (Non-Survey)

Assumptions:
- Equal factor productivity and consumption shares between region-nation
- Equal production mix
- Nation neither imports or exports the good in net terms (underestimate the base)
- All local demand is met first by local production (no cross-hauling)

What indicator to use?
- Employment (most used): issue of equal weight upon full/part-time, disregards productivity and wage differentials among sectors
- Income or Value Added: more difficult to have at regional level

Location Quotients

overestimate the multiplier
## The Two-Sector Model

### Location Quotients: Example

<table>
<thead>
<tr>
<th>Sector</th>
<th>Regional Employment</th>
<th>%</th>
<th>National Employment</th>
<th>%</th>
<th>Location Quotient</th>
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<tr>
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<td>50</td>
<td>5%</td>
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<td>16%</td>
<td>1.25</td>
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<td>Total</td>
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</table>
The Two-Sector Model
Location Quotients: Example

• Estimating the LQ for sector 1:

\[ LQ_i^r = \left( \frac{x_i^r}{x^r} \right) / \left( \frac{x_i^{nation}}{x^{nation}} \right) = \frac{0.1}{0.05} = 2 \]

• If LQ > 1: part of the sector’s production is exported

• To estimate exports, we assume a location quotient of 1.0 implies “self-sufficiency”; any employment above this is export

• For sector 1, self-sufficiency would be 5% of the region’s total (similar to national share); hence, of the region’s 10% of total employment in sector 1, 5% serves the regional market and 5% serves the export market
# The Two-Sector Model

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<th>%</th>
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The Two-Sector Model
Location Quotients: Example

How do we estimate the multiplier?

• Given the share of non-basic/total:

\[ \alpha = \frac{L_n}{L_T} = \frac{71}{100} = 0.71 \]

• Hence, the multiplier becomes:

\[ \frac{1}{1 - \alpha} = 3.45 \]

• Each export job generates another 2.45 jobs for a total 3.45 jobs in the region
The Two-Sector Model Implementation: Identifying Basic Activities (Non-Survey)

Minimum Requirements Technique

➢ Compares regional employment structure with similar size regions, identifies the one with the least share of employment in an industry and it becomes the **benchmark**

$$LQ_i^r = \left( \frac{x_i^r}{x^r} \right) \Bigg/ \left( \frac{x_i^{benchmark}}{x_{benchmark}} \right)$$

➢ Assumptions:
  ➢ Similar as before

➢ Issues:
  ➢ Sample size impacts sector identification
  ➢ If detail data is used, may reduce local needs to near zero
The Two-Sector Model
Connection to Macroeconomic Variables

- Export-led Keynesian Model

\[ Y = C + G + I + X - M \]

where:

\[ X = \bar{X} \text{ (exogenous)} \]

\[ C = cY \text{ (marginal propensity to consume)} \]

\[ M = mY \text{ (marginal propensity to import)} \]

\[ Y = \frac{1}{1 - (c - m)} \bar{X} \]
The Two-Sector Model

Summary

• Important model because it provides a useful mechanism to help understand how a region grows

• Idea of export and local-oriented sectors is useful in considering development strategies

• The aggregate nature of the model limits applications
The Multi-Sectors Model

Input-Output Analysis
The Multi-Sectors Model
Input-Output Introduction

- Proposed by Wassily Leontief in the 1930s
  Nobel Prize in Economics in 1973 – died in February 1999 at age 92

- Provides a complete vision of industrial interdependency in a region through the analysis of interindustrial flows
  → extends ideas of the economic base model by disaggregating production into a set of sectors

- Linear system of equations, representing the production function of each sector
- Reveals indirect impacts via industrial chains
- Derived from the National Accounts, satisfies macroeconomic identities
The Multi-Sectors Model

**Input-Output Table**

- “Picture” of the economy for a given year → show all industrial monetary flows for the region
- The Input-Output Table is basically an accounting system (i.e., double entry (inputs/outputs))
- Preserves macroeconomic identities
The Multi-Sectors Model
Input-Output Table

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</tbody>
</table>
The Multi-Sectors Model
Input-Output Table

- Macroeconomic Identity:

\[
\begin{align*}
\text{Sale Structure} & \quad \text{Input Structure} \\
1 \quad X &= Z + C + G + I + E & 2 \quad X &= Z + VA + M \\

Z + C + G + I + E &= Z + M + VA \\
C + G + I + (E - M) &= VA \\
\text{GDP (expenditure approach)} & \quad \text{GDP (income approach)}
\end{align*}
\]
The Multi-Sectors Model

Input-Output Model

- Industries have two roles: purchase inputs / sell output → interindustrial flows
  - E.g.: Bakery: buys flour to produce bread, sells bread to restaurants / you
- Goods purchased for final consumption → final demand
  - E.g.: You buy bread for breakfast / You buy flour to bake a cake at home
- Hence, the basic I-O model:

\[ x_i = y_i \]

\[ x_i = z_{i1} + \cdots + z_{in} + f_i \]

\[ x = Z_i + f \]
The Multi-Sectors Model
Input-Output Model

• Assumption:

    *Interindustry flows from industry $i$ to $j$ depend entirely on the total output of sector $j$ for that same period*

It takes the form: $a_{ij} = \frac{Z_{ij}}{x_j}$

**technical coefficient (direct input coefficient) → fixed proportion**

For example: $a_{ij} = \frac{\text{value of flour bought by bakeries last year}}{\text{value of bakery production last year}}$
The Multi-Sectors Model
Input-Output Model

• This definition implies:

\[ x_j = \frac{z_{1j}}{a_{1j}} = \frac{z_{2j}}{a_{2j}} = \ldots = \frac{z_{jj}}{a_{jj}} = \ldots = \frac{z_{nj}}{a_{nj}} \]

• So, the production function:

\[ x_j = \min \left\{ \frac{z_{1j}}{a_{1j}}, \frac{z_{2j}}{a_{2j}}, \ldots, \frac{z_{jj}}{a_{jj}}, \ldots, \frac{z_{nj}}{a_{nj}} \right\} \]

Perfect complements

→ constant returns to scale

Leontief Production Function
The Multi-Sectors Model

Input-Output Model

\[ x_1 = a_{11}x_1 + \cdots + a_{1i}x_i + \cdots + a_{1n}x_n + f_1 \]
\[ \vdots \]
\[ x_i = a_{i1}x_1 + \cdots + a_{ii}x_i + \cdots + a_{in}x_n + f_i \]
\[ \vdots \]
\[ x_n = a_{n1}x_1 + \cdots + a_{nj}x_j + \cdots + a_{nn}x_n + f_n \]

\[ \mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{f} \]

Final demand exogenous

\[ (\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{f} \]

\[ \mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} \]

Leontief Inverse

Total Requirement Matrix (\(\mathbf{B}\))
The Multi-Sectors Model
Input-Output Model

Technical Coefficients Matrix
(Direct Coefficients Matrix)

$$A = Z(\hat{x})^{-1}$$

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Output

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The Multi-Sectors Model
Input-Output Model

- Ripple effects:

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E.g.: Increase Mfg production in $1:

Total Impact: $1.74
Direct: $1.00
Indirect: $0.74

Addition

$1.00
$0.44
$0.18
$0.07
The Multi-Sectors Model
Input-Output Model

\[
A = Z(\hat{x})^{-1}
\]

\[
x = Zi + f \quad \Rightarrow \quad x = Ax + f \quad \Rightarrow \quad x = (I - A)^{-1}f
\]

Leontief Inverse
Total Requirement Matrix

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<tbody>
<tr>
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<td>Serv.</td>
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</table>

**Type I Multipliers**

Direct Effect: 1.00 1.00 1.00
Indirect Effect: 0.67 0.74 0.70

reflect the degree to which a sector is dependent on other sectors in the region for its inputs and as a source of consumption for its products.
The Multi-Sectors Model

**Input-Output Multiplier vs Economic Base Multiplier**

- Economic Base Model
  - Single multiplier: \( L_T = \frac{1}{1 - \alpha} \bar{L}_b \)

- Input-Output Model
  - Matrix of multipliers: \( x = (I - A)^{-1} f \)
  - Ripple Effect: \((I - A)^{-1} = I + A + A^2 + A^3 + A^4 + \cdots\)
The Multi-Sectors Model

Input-Output Model - Notes

• It would be incorrect to assume that a sector’s importance in the economy is directly related to the size of the multiplier:
  • While true in part, a sector with a large volume of production but a modest multiplier may generate a greater volume of activity in the region than the sector with the largest multiplier but a smaller volume of production.

• Multipliers vary across regions: A small regional economy, with a modest representation of industry, may not be able to provide all the necessary inputs required by local industry → Thus, there will be considerable importation of inputs (sometimes referred to as leakages):
  • In general, the larger the value of the imports, the lower the value of the multiplier.
  • We would expect multipliers to decrease as we move from the an individual country as a whole to a region, a metropolitan region and finally to a county.
The Multi-Sectors Model
Input-Output Model (closed w.r.t. HH)

- We can capture not only interindustrial linkages but also the industrial linkage to households via wages and consumption:

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<td>20</td>
<td>20</td>
<td>2</td>
<td></td>
<td></td>
<td>42</td>
</tr>
<tr>
<td>Capital</td>
<td>10</td>
<td>20</td>
<td>18</td>
<td></td>
<td></td>
<td>48</td>
</tr>
<tr>
<td>Import</td>
<td>40</td>
<td>50</td>
<td>30</td>
<td></td>
<td></td>
<td>120</td>
</tr>
<tr>
<td>Output</td>
<td>115</td>
<td>160</td>
<td>85</td>
<td>90</td>
<td>120</td>
<td></td>
</tr>
</tbody>
</table>
The Multi-Sectors Model
Input-Output Model (closed w.r.t. HH)

• Endogenize households:
  • $\mathbf{h}_R = [a_{n+1,1}, \ldots, a_{n+1,i}, \ldots, a_{n+1,n}]$: labor input coefficients
  • $\mathbf{h}_C' = [a_{1,n+1}, \ldots, a_{i,n+1}, \ldots, a_{n,n+1}]$: households’ consumption coefficients
  • $h$: labor employed by households

• Hence: $\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{h}_C \\ \mathbf{h}_R & h \end{bmatrix}$ \quad $\bar{\mathbf{x}} = \begin{bmatrix} \mathbf{x} \\ x_{n+1} \end{bmatrix}$ \quad $\bar{\mathbf{f}} = \begin{bmatrix} \mathbf{f} \\ f_{n+1} \end{bmatrix}$

• And $\bar{\mathbf{x}} = (\mathbf{I} - \bar{\mathbf{A}})^{-1} \bar{\mathbf{f}}$ \quad $\Rightarrow$ \quad $\begin{bmatrix} \mathbf{x} \\ x_{n+1} \end{bmatrix} = \begin{bmatrix} \mathbf{I} - \mathbf{A} & -\mathbf{h}_C \\ -\mathbf{h}_R & 1 - h \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{f} \\ f_{n+1} \end{bmatrix}$
## The Multi-Sectors Model

Input-Output Model (closed w.r.t. HH)

- From the previous example:

### Leontief Inverse

<table>
<thead>
<tr>
<th></th>
<th>Agr.</th>
<th>Mfg</th>
<th>Serv.</th>
<th>HH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agr.</td>
<td>1.20</td>
<td>0.31</td>
<td>0.34</td>
<td>0.63</td>
</tr>
<tr>
<td>Mfg</td>
<td>0.27</td>
<td>1.23</td>
<td>0.22</td>
<td>0.69</td>
</tr>
<tr>
<td>Serv.</td>
<td>0.20</td>
<td>0.21</td>
<td>1.14</td>
<td>0.75</td>
</tr>
<tr>
<td>HH</td>
<td>0.30</td>
<td>0.26</td>
<td>0.14</td>
<td>1.21</td>
</tr>
</tbody>
</table>

### Type II Multipliers

- Direct Effect: 1.00
- Indirect Effect: 0.67
- Induced Effect: 0.81

### Type I Multipliers

- Direct Effect: 1.00
- Indirect Effect: 0.74
- Induced Effect: 0.70

The output includes a table that represents the Leontief Inverse matrix for different sectors and their multipliers. The table is organized to show the direct, indirect, and induced effects on each sector.
The Multi-Sectors Model
Analyzing the Economy – Linkages in I-O Models

- Key Sector Identification: those with above average impact on the regional economy
  - Recall: sectors are both purchasers and sellers in the economy
  - Sectors with above backward and forward linkages → **key sector**

\[
BL_j = \frac{(\sum_i B_{ij}/n)}{(\sum_i \sum_j B_{ij}/n^2)} = \frac{n \sum_i B_{ij}}{\sum_i \sum_j B_{ij}}
\]

Index of the Power of Dispersion (Backward Linkages)

\[
FL_i = \frac{(\sum_j B_{ij}/n)}{(\sum_i \sum_j B_{ij}/n^2)} = \frac{n \sum_j B_{ij}}{\sum_i \sum_j B_{ij}}
\]

Index of Sensibility of Dispersion (Forward Linkages)

\[BL_j > 1 \text{ (} FL_i > 1 \text{) indicates that sector } j \text{ (} i \text{) has above average backward (forward) linkages}\]
The Multi-Sectors Model
Analyzing the Economy – Linkages in I-O Models

- Example:

<table>
<thead>
<tr>
<th>Sector</th>
<th>Sector 1</th>
<th>Sector 2</th>
<th>Sector 3</th>
<th>Sector 4</th>
<th>Sector 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
<td>1.33</td>
<td>0.05</td>
<td>0.18</td>
<td>0.15</td>
<td>0.02</td>
</tr>
<tr>
<td>Sector 2</td>
<td>0.23</td>
<td>1.17</td>
<td>0.3</td>
<td>0.5</td>
<td>0.04</td>
</tr>
<tr>
<td>Sector 3</td>
<td>0.4</td>
<td>0.36</td>
<td>1.41</td>
<td>0.82</td>
<td>0.13</td>
</tr>
<tr>
<td>Sector 4</td>
<td>0.58</td>
<td>0.19</td>
<td>0.61</td>
<td>1.38</td>
<td>0.09</td>
</tr>
<tr>
<td>Sector 5</td>
<td>0.31</td>
<td>0.41</td>
<td>0.48</td>
<td>0.38</td>
<td>1.09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Row Multiplier</th>
<th>Forward Linkages</th>
<th>Rank-Size Hierarchy of Forward Linkages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.73</td>
<td>0.69</td>
<td>V</td>
</tr>
<tr>
<td>2.24</td>
<td>0.89</td>
<td>IV</td>
</tr>
<tr>
<td>3.12</td>
<td>1.24</td>
<td>I</td>
</tr>
<tr>
<td>2.85</td>
<td>1.13</td>
<td>II</td>
</tr>
<tr>
<td>2.67</td>
<td>1.06</td>
<td>III</td>
</tr>
</tbody>
</table>

| Column Multiplier | 2.85 | 2.18 | 2.98 | 3.23 | 1.37 |
| Backward Linkages | 1.13 | 0.86 | 1.18 | 1.28 | 0.54 |
| Rank-Size Hierarchy of Backward Linkages | III | IV | II | I | V |
The Multi-Sectors Model
Analyzing the Economy – Linkages in I-O Models

• Classification of Backward and Forward Linkage Results

<table>
<thead>
<tr>
<th>Backward Linkages</th>
<th>Low ( &lt; 1)</th>
<th>High ( &gt; 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Generally independent</td>
<td>Dependent on interindustrial demand</td>
</tr>
<tr>
<td></td>
<td>Dependent on interindustrial supply</td>
<td>Generally dependent</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Forward Linkages</th>
<th>Low ( &lt; 1)</th>
<th>High ( &gt; 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Generally independent</td>
<td>Dependent on interindustrial demand</td>
</tr>
<tr>
<td></td>
<td>Dependent on interindustrial supply</td>
<td>Generally dependent</td>
</tr>
</tbody>
</table>

• In the example:
  • Generally independent: Sector 2
  • Generally dependent: Sector 3, Sector 4
  • Dependent on interindustrial demand: Sector 5
  • Dependent on interindustrial supply: Sector 1
The Multi-Sectors Model

Interregional Input-Output Model

- Interregional Framework (Walter Isard, 1960)
  - Single-region model: truncate circular flow since each region is “disconnected” from others
  - Multi-region model: accounts for *interregional feedbacks*

- Increased Demand for Washington Aircrafts
- Increased Output of Washington Sectors
- Increased Output of Connecticut Sectors

**Intrarregional** Effects
- (in WA)
- (in CT)

**Interregional** Effects
The Multi-Sectors Model
Interregional Input-Output Model

- Consider 2 regions: Region 1 (n sectors) and Region 2 (m sectors)

In matrix notation:

\[ Z = \begin{bmatrix} Z^{11} & Z^{12} \\ Z^{21} & Z^{22} \end{bmatrix}, \quad x = \begin{bmatrix} x^1 \\ x^2 \end{bmatrix}, \quad f = \begin{bmatrix} f^1 \\ f^2 \end{bmatrix} \]

\[ A = \begin{bmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{bmatrix} \]
The Multi-Sectors Model
Interregional Input-Output Model

• Then,

\[
\begin{bmatrix}
I & 0 \\
0 & I
\end{bmatrix} - \begin{bmatrix}
A^{11} & A^{12} \\
A^{21} & A^{22}
\end{bmatrix} \begin{bmatrix}
x^1 \\
x^2
\end{bmatrix} = \begin{bmatrix}
f^1 \\
f^2
\end{bmatrix}
\]

or

\[
(I - A^{11})x^1 - A^{12}x^2 = f^1 \\
-A^{21}x^1 + (I - A^{22})x^2 = f^2
\]

• Single-region vs interregional impacts:

Assume \( f^2 = 0 \) \( \Rightarrow x^2 = (I - A^{22})^{-1}A^{21}x^1 \)

\[
(I - A^{11})x^1 - A^{12}(I - A^{22})^{-1}A^{21}x^1 = f^1
\]

- Single region effect
- Direct + indirect impacts on 2 to supply 1
- Additional sales from 1 → 2 required
- Economic self-influence transactions of one of the regions through the other region

Sectors M \( \rightarrow \) Sectors S

Sectors S \( \rightarrow \) Sectors M
The Multi-Sectors Model

Interregional Input-Output Model

- Finally, notice that
  - Single-region: \( x^1 = (I - A^{11})^{-1}f^1 \)
  - Multi-region: \( x^1 = (I - A^{11} - A^{12}(I - A^{22})^{-1}A^{21})^{-1}f^1 \)

\[
(I - A^{11})^{-1} < (I - A^{11} - A^{12}(I - A^{22})^{-1}A^{21})^{-1}
\]

- Using Schur’s formula, the Leontief Inverse can be written as:

\[
B = \begin{bmatrix}
B^{11} & B^{1}A^{12}B^{22} \\
B^{2}A^{21}B^{11} & B^{22}
\end{bmatrix}
\]

where

\[
B^{1} = (I - A^{11})^{-1} \quad B^{11} = (I - A^{11} - A^{12}(I - A^{22})^{-1}A^{21})^{-1}
\]

\[
B^{2} = (I - A^{22})^{-1} \quad B^{22} = (I - A^{22} - A^{21}(I - A^{11})^{-1}A^{12})^{-1}
\]

How large is this difference? Depends on the strength of the interregional linkages (on how self-sufficient the regions are).
The Multi-Sectors Model
National Data

• Official I-O tables are usually released in 3-5 years intervals (benchmark tables)

• Or can be estimated from the National Accounts
  • More precisely from the Use and Make Tables
  • Remember: Tables are in basic prices (= purchase prices – taxes – transp. margin – trade margin)

• Freely available data:
  • World Input-Output Database (1995-2011) : interregional (40 countries)
  • EORA (1990-2012) : multi-regional (187 countries)
  • OECD (1995-2011) : national level (all OCDE + 27 countries)
The Multi-Sectors Model
Regional Data

- Official Regional I-O Tables are usually unavailable (surveys are expensive!)
- Non-survey method of estimation (common practice):

  - Location Quotients to derive *intraregional* tables by scaling down the national table
  - Gravity Model to derive *interregional* flows
  - RAS Technique to balance the table

See Miller and Blair (2009) Chapter 8 for full description of the procedure.
The Multi-Sectors Model

Other Models

• From input-output we can move to social accounting models in which attention is paid to income distribution and the role of institutions (such as government) and transfers of non wage and salary kind (dividends, pensions, taxes etc.)

• Other models link input-output with linear programming, econometric and general equilibrium formulations – with some models containing 2 million equations!
The Multi-Sectors Model

Summary

- Both input-output and economic base model share a demand-led structure.
- The economic base model is more suitable to smaller, the input-output to larger, more sophisticated economies.
- Models are important inputs into decision-making but they require the analyst to understand their uses and their limitations.
Further Reading
Further Reading

- Economic Base Model:
  - See Tiebout-North discussion on the model:
Further Reading

• Location Quotients
Further Reading

- Input-Output Analysis