

# A Mathematical Interpretation of Human Body Movement

Mohammad Amanzadeh

July 1, 2019

MA: Edits as  
stipulated by  
Examiners, 26jun19.

## Abstract

This proposal explains a machine learning program that is capable of learning the motions of a human user by animating a puppet that mimics the human body structure, and recognizing the physical properties of the motion, such as the expressed intention and mood, to translate the expressions into different forms of visualization and sound. The program includes an *application programming interface* (API) that animates a teacher puppet from active user inputs, and simultaneously teaches a student puppet to mimic and learn variants of the teacher's dance. The API could be used to create an interactive human-computer application to help and encourage the rehabilitation of patients afflicted with motor control impediments, such as the cerebral palsy, through performing physical activities that engage the user's creativity, such as dance, music performance, and drawing. The proposed API will also measure movement of a human body relative to the computer screen, learn and recognize movement patterns, and provides realtime update about the progression and variation of a motion with respect to the learned/recognized pattern. The API would be used to develop augmented reality environments that support rehabilitation and creating art.

## 1 Introduction

The research I plan to do for my PhD consist in developing an application programming interface (API) that learns to recognize intention and meaning of movements from active user inputs. The program mimics the structure and possible motion of the human body by animating two copies, the *teacher* and the *student*, of a puppet based on Sherwin Tam's *illiJoe realtime interactive computer animation* (RTICA) project, implementing

Jim Blinn’s Blobbyman [1] in Prof. Francis’s Math198 “Hypergraphics” course<sup>1</sup>.

I translated illiJoe to Javascript and improved its interface to have clockwork animation while you can animate it simultaneously by a mouse. The following link is a version of the RTICA that I developed as part of my qualification exam last August<sup>2</sup>.

The application programming interface (API) interprets active user input to teach a machine-learning algorithm to recognize intention and mood by animating an articulated humanoid puppet. The user will recognize the failure or success of this mimicry, and modify the learning set when appropriate.

Towards this end I have been working on a proof-of-concept RTICA using Jim Blinn’s scene-graph animated puppets<sup>3</sup>. For example, an initial version has two identically configured bobbies, the “teacher” and her “student”. Initially, she just undulates according to nonlinear, gravity-like forces on each joint. But the student can only use simple harmonic motion (sinusoidal oscillations) about two angles in each joint. He thus becomes a platform for various machine learning strategies to mimic his teacher as best as he can, but lacking the ability to reproduce them exactly.

The user observes both to judge their success and interactively contaminates the clarity of the teacher’s motion (e.g. suppressed one dimension, as would be the case in the student facing a video of a human dancer.) Obviously, each component will improve in “realism” for the teacher, such as a video of a human dancer. And for the student, there will be persuasive, robust learning algorithm.

In an earlier work [2] we studied how the beat-pattern movement of a conductor communicates the time progression and the articulation style of a music performance, such as the quality of performing a sostenuto vs. staccato sound. The study found that the velocity and acceleration of the motion during a beat interval could be a good indicator and predictor of how a motion is performed. This information could be the input of a classifier that translates movement to some language, such as the *Laban Movement Analysis* (LMA), or to be used in producing visualization and sounds that augmenting movement qualities.

## 2 The Teacher Puppet

A simple human movement, such as waving a hand, is composed of the motion of sequence of joints and bones leading to the hand. The sequence

---

<sup>1</sup><http://new.math.uiuc.edu/math198/198.96/stam/stam.html>

<sup>2</sup><http://http://new.math.uiuc.edu/infoweb17/ijoe6aug18.html>

<sup>3</sup><https://computeranimationhistory-cgi.jimdo.com/blobby-man-1979/>

constitute a so-called *kinematic chain* more commonly known as a *mechanical linkage* [3]. It can be specified by the lengths of the bones and the orientations of the joints in the world coordinates.

But, as Blinn points out, [1] it is more conveniently expressed in the coordinate system of the previous joint. Thus a joint can be represented by a Euclidean motion consisting of a rotation  $U$  and a displacement  $m$ , which is traditionally<sup>4</sup> represented in homogeneous coordinates as a 4x4 matrix with a 3x3 block for rotation  $U$  a 3x1 column vector for displacement  $m$  and a bottom row of zeros ending in a 1.

$$P = \left[ \begin{array}{c|c} U & m \\ \hline 0 & 1 \end{array} \right]$$

Thus the place  $P_{\text{handinworld}}$  of the hand in world coordinates is given by the composition of the four Euclidean motions, easily computed by the matrix product of the places.

$$P_{\text{handinworld}} = P_{\text{body}} P_{\text{shoulder}} P_{\text{elbow}} P_{\text{wrist}}$$

where  $P_{\text{wrist}}$  is the place of the hand in the wrist coordinate system, etc.

To display the hand, the vertices of the hand,  $X_{\text{hand}}$ , are expressed in homogeneous coordinates and transformed by current place of the hand in world coordinates as

$$X_{\text{handinworld}} = P_{\text{handinworld}} X_{\text{hand}} = U_{\text{world}} X_{\text{hand}} + m_{\text{world}}$$

We studied mechanics of a pendulum as a prototype experiment to understand how the motion of a pendulum expresses physical properties of the pendulum, its initial position and initial movement, as it expands in the gravitational field. The reason to study a pendulum is its simple mechanism and similarity of its motion to human body movements, because the motion is periodic and it expresses a property or condition as it expands in the space. Thus as pointed out by Ruecker et al. [4] we can expect that the knowledge gained from this prototyping can be applied back to implement the original idea of developing a teacher RTICA that teaches a student RTICA to express an aesthetic property in a motion.

## 2.1 The Simple Pendulum

A simple pendulum is a mechanical idealization of the real pendulum physicists have explored since Galileo in the sixteenth century. Consisting of a rigid but weightless rod and a point mass at one end of it, which the other end is free to rotate about a fixed base point in space, it is acted on by

---

<sup>4</sup>Other representations, for instance using quaternions for the rotations, are in use.

gravity alone as it is released from an initial position other than its rest position. It will subsequently oscillate in the plane through the base point and perpendicular to the cross-product of the gravity vector and displacement vector of the rod.

Translating this qualitative analysis into of Newtonian gravitational mechanics yields the classical pendulum equation

$$\ddot{\theta} + \sin(\theta) = 0$$

where  $\theta$  is the angle the pendulum makes from the vertical, and  $\ddot{\theta}$  is its angular acceleration. We have normalized gravitation, mass and length of the pendulum to one for simplicity.

As such, this differential equation of the physical pendulum does not have a closed form solution. For small  $\theta$  the approximation  $\sin \theta \approx \theta$  linearizes the system to

$$\ddot{\theta} + \theta = 0$$

whose well known solution is called *harmonic oscillation*.

A vastly over simplified model with a physical pendulum as teacher, but a linear (harmonic) pendulum as student already provides the student an impossible motion to mimic. The harmonic pendulum is unable to copy the exaggerated motion of a physical pendulum that “goes over the top” and rotates instead of swinging back and forth. This is the first example of a teacher’s “emotion” which the student must “learn” by continuously changing his three parameters.

### 2.1.1 The Physical Pendulum

We use Newton’s laws of motion to derive the motion of a simple gravity pendulum.

According to the first law of motion a pendulum in zero gravity is either at rest or it rotates at a constant speed  $\dot{\theta} = \psi$ . The second and third laws explain that the pendulum accelerates proportional the gravity pull  $g$  and in opposite direction  $\dot{\psi} \propto -g$ . Because of the superposition principle, a gravity vector oriented at  $\theta$  degrees in direction of the pendulum, pulls the base along the rod with magnitude  $g \cos(\theta)$  and it pulls the weight perpendicular to the rod with magnitude  $g \sin(\theta)$ . Assuming that the base does not move and the rod length is equal to  $g$ , the pendulum’s motion is derived by the differential equation

$$\ddot{\theta} + \sin(\theta) = 0$$

### 2.1.2 Simple harmonic motion

Simple harmonic motion approximates the pendulum’s oscillatory motion when  $\theta$  is small enough and thus  $\sin(\theta) \approx \theta$ . This assumption simplifies

the pendulum's equation to

$$\ddot{\theta} + \theta = 0$$

The motion of a pendulum released at angle  $\theta$  is derived by solving the system of differential equations

$$\begin{cases} \dot{\theta} = \psi \\ \dot{\psi} = -\theta \end{cases}$$

The closed form solution to this equation assuming that the pendulum starts at angle  $\theta_0$  and oscillates with an amplitude  $A$ , and frequency  $\lambda$ , is given by

$$\theta(t) = A \sin(\lambda t + \phi)$$

where  $\phi = \arcsin(\theta_0/A)$ .

## 2.2 The compound pendulum

We define a compound pendulum as a second harmonic pendulum whose base is located at the weight of a prior simple harmonic pendulum, which can be extended to a mechanical linkage of arbitrary length. Because of the simple harmonic pendulum assumption the motion of the parents weight does not affect the motion of the child's base in the coordinates of the parent. Thus, unlike a physical double pendulum, which has chaotic motion, our simplified compound pendulum produces harmonic motion by superimposing the oscillation of a lower pendulum over the oscillation of its previous pendulum.

In space, a pair of pendulums could have three configurations depending on the degrees of freedom of the base plane, that specifies whether the base coordinates preserves its orientation in world coordinates, or it can rotate about its three axes by the motion of a previous pendulum.

Thus we approximate the human body (to the first degree of abstraction) as a *tree graph*<sup>5</sup> of compound pendulums, each bone of which can rotate in about two orthogonal axes in the coordinate system of the parent joint. The third rotation about the axis of the bone itself, for example rotating the hand about the axis of the forearm, can be effected by rapid successions of two rotations about transverse axes, when required. This process is sometimes referred to as "milling".

## 3 The Student Puppet

The student puppet learns the animation of a teacher puppet by actively reconstructing the motion of the teacher's linkages given by the motion

---

<sup>5</sup>Other commonly used terms for this are hierarchy, scene graph, articulation, etc.

of her joints. The student might observe the exact 3D position of the teacher's joints, and also it is possible that some dimension, such as depth, be partially available, in which case the student estimates the missing dimension using *inverse kinematics*[5].

## 4 Improving the Teacher

The stages of improving the teacher from a clock-work automaton to ever more realistic and emotionally expressive teachers begins with studying how motions of the human body communicate different expressions.

Humans perceive the outside world through various senses. The brain's perceptual system continuously interprets the world's stimulations to the sensory organs to create an internal understanding of the outside world [6]. Movement is an ability of the human body that makes possible to interact with its proximal (near) world or communicate to a distal (distant) world.

### 4.1 Interpreting the beat-pattern motions of an orchestra conductor

Orchestral Conducting provides a structured language to communicate different musical expressions using body movements. Using the beat-pattern gesture, a conductor communicates the time progression and the articulation style of a music, such as the quality of performing sostenuto or staccato, to the orchestra. As explained by Lee et al. [2] it is possible to distinguish the expressed articulation style of a beat-pattern by comparing the summary statistics of the velocity and acceleration of the motion in a beat interval with a baseline.

## 5 Possible Applications of the API

The construction of an API can be justified only if it leads to useful applications. For this dissertation I propose to apply my API to some simple examples involving healthy human motor control such as the teacher puppet executing a pirouette to the student puppet based on motion capture. This could be useful to gain insight why some dance motions are harder to teach than others.

However, a future application of the API might be to build assistive technologies for rehabilitation of motor impaired individuals within *virtual environments* (VE), which would like to utilize the attributes of *virtual reality* (VR) technology along with the art and science of movement, to design physical exercises that incorporate the creativity of patients within

an *aesthetically resonant virtual environment* (ARVE) to immerse and engage them in the rehabilitation process[7].

A VE is a simulation of a real-world environment or activity that is experienced through the human-machine interface, such as the visual displays, sound system, haptic devices, and so on. The participant uses the senses to obtain information about the virtual environment through the interface. This input is combined with the natural sensory input received from the real environment and create a hybrid input to the *central nervous system* (CNS), which could include the participant as if the environment were real [8].

Computer games and VR are widely used for rehabilitation of individuals with different motor impairment. According to Holden [8] people with disability are capable of motor learning within a virtual environment and in most cases, they can transfer the learned movement to real-world equivalent of the motor task. Although, one should consider that VR is a technology, not a treatment, and its success in rehabilitation largely depends on the scientific rationale behind motor learning and detail of the specific motor impairment.

Repeated practice and incremental success is an important concept in motor learning. An advantage of using VR for motor learning is to allow the patient to repeat a practice, and simultaneously, the system provides feedback about the quality of performance.

Feedback is the most discussed attribute of VR that is utilized in motor learning. However, there are also other factors that affect the user engagement in a virtual environment. According to O'Brien and Toms [9], “engagement is a quality of user experience characterized by attributes of challenge, positive affect, endurability, aesthetic and sensory appeal, attention, feedback, variety/novelty, interactivity, and perceived user control”.

VE provides the foundation for implementing many aspects of an engaging experience, however, an engaging technology might not necessarily trigger the sense of “flow”. O'Brien and Toms[9] referred to the *flow* as a condition in which someone is so immersed in an activity that “nothing else seems to matter”. They explained that engagement and flow share some common attributes, the sense of flow specifically requires intrinsic motivation, sustained focus, and loss of awareness of the outside world. As also suggested by O'Brien and Toms, an aesthetic experience includes many characteristics of flow, for example, playing music, dancing, or painting.

To maximize the therapeutic effects of a system for rehabilitation, Vuong [10] suggests that one should consider the symptoms and conditions that are specific to the disease and its development stage, such as the clinical techniques used for rehabilitation of the specific disease and its neural science base. These considerations could inform about the design and flow of the visual and auditory information in the system. In addition,

it is vital to provide accurate feedback in response to the motion, because asynchronous feedback does not communicate the sense of control to the user, which would result in frustration and disengagement of the user. Another important design requirement is the ability to automatically or semi-automatically calibrate the system to match the physical specifications of a particular participant[11].

Embodiment is an interdisciplinary and theoretical approach for empirical research in the field of *art therapy* [12], which explains the body as an “object” that is perceived from both inside and outside. Embodiment investigates the mutual influence of the body as a living and moving organism, on one hand, and the *Perception, Cognition, Action* loop, on the other hand, to discuss the interactions and expressive functions of an individual. Therefore, movement can directly influence the cognition and affect. For example, smooth rhythms in handshake brings a more positive effect, and represents an extroverted and agreeable personality, unlike a handshake with sharp rhythm [12].

All these techniques utilized some form of feedback to engage the patients in the exercise and guide their movement. The game based systems mostly focused on feedback about goal achievement, and used positive scores to reward the patients. In addition, some systems used real-time audiovisual feedback (or cues) to augment the patient’s sensorimotor and help them to anticipate and correct dynamics of their movement. This type feedback was mostly used in ARVEs, although, an ARVE could also incorporate the features of a serious game.

The physical exercises use real-world objects to cue a patient about his/her movement. For example, a walk used vertical bars placed at equal distances along a walkway to assist patients to stay conscious about their walking pace.

My understanding is that the most important activity for rehabilitation is the physical exercise, while the role of VR is to make the exercise more engaging and assist the patient to be aware of his/her body movement through the virtual cues and real-time feedback. In addition, the activities that require creativity, such as painting, dancing, performing music, or solving games, could provide additional rewards to the patient and enhance the sense of immersion into Virtual Reality.

Currently I am studying the literature around the physical exercises for rehabilitation to come up with ideas about the exercises that could be implemented within the ARVE. The following quotation from [13] will be elaborated in a later revision of this proposal.

Individuals with *cerebral palsy* (CP) have limitations with posture and movement that reduce participation in physical and social activities.

1. CP is diagnosed in childhood but is a lifelong condition; with

increasing life expectancy, most people with CP now live into adulthood. However, balance, strength, and endurance appear to decline with age, thereby reducing activity.

2. Along with motor deficits, coexisting conditions commonly occur with CP, including sensory deficits, epilepsy, intellectual impairment, learning problems, attention deficit, autism, and musculoskeletal misalignment. These deficits impact the child over time.
3. With age, children and adults with CP become increasingly sedentary, further reducing social engagement and participation.
4. Activities and opportunities for therapeutic movement and rehabilitation, such as physical therapy, decline over time for children with CP.
5. Community resources and accessibility vary, and often are limited within healthcare systems.
6. Children with CP face shifts in priorities with increasing responsibility in school and greater importance placed on relationships and socializing as they become adolescents. Dance classes and moving to music provide individuals with CP opportunities to explore movement in environments that can address associated impairments. Dance “a creative and expressive art” generally involves the performance of movement to music. Incorporating dance as an art form into rehabilitation has capacity to transcend traditional barriers in therapy that differentially focus on impairments and limitations. Dance enables opportunities for engaging in a social activity, while providing therapeutic benefit.

7. *Rhythmic auditory stimulation* (RAS), which also includes movement and rhythm, has a greater emphasis on synchronization of gait to a steady external rhythm, which may include melody, and does not focus on the performative capacity of gait.<sup>8</sup> Participation in artistic endeavors may not only provide a motivating environment, but also contribute to enhanced recovery through opportunities for enjoyable creative expression.

Studies showed preliminary evidence of the benefits of dance and RAS on body functions, particularly balance, gait, walking, and cardiorespiratory fitness for individuals with CP. Research gaps are evident across all domains of the ICF, particularly in the participation and environment domains.

## References

- [1] Jim Blinn. Nested transformations and blobby man. *IEEE Computer Graphics And Applications*, 7, 1987.
- [2] Kyungho Lee, Michael J Junokas, Mohammad Amanzadeh, and Guy E Garnett. Exploratory analysis on expressions in two different 4/4 beat patterns. In *ICMC*, 2015.
- [3] Mechanical linkage wikipedia. [https://en.wikipedia.org/wiki/Linkage\\_\(mechanical\)](https://en.wikipedia.org/wiki/Linkage_(mechanical)).
- [4] Stan Ruecker, Nadine Adelaar, Susan Brown, Teresa Dobson, Ruth Knechtel, Susan Liepert, Andrew MacDonald, Ernesto Peña, Milena Radzikowska, Geoff G Roeder, et al. Academic prototyping as a method of knowledge production: The case of the dynamic table of contexts. *Scholarly and Research Communication*, 2014.
- [5] Inverse kinematics wikipedia. [https://en.wikipedia.org/wiki/Inverse\\_kinematics](https://en.wikipedia.org/wiki/Inverse_kinematics).
- [6] Perception wikipedia. <https://en.wikipedia.org/wiki/Perception>.
- [7] Anthony Lewis-Brooks and Stefan Hasselblad. Creating aesthetically resonant environments for the handicapped, elderly and rehabilitation: Sweden. *International Journal on Disability and Human Development*, 4(4):285–294, 2005.
- [8] Maureen K Holden. Virtual environments for motor rehabilitation: review. *Cyberpsychology & behavior*, 8(3):187–211, 2005.
- [9] Heather L O’Brien and Elaine G Toms. What is user engagement? a conceptual framework for defining user engagement with technology. *Journal of the American Society for Information Science and Technology*, 59(6):938–955, 2008.
- [10] Catherine H Vuong, Todd Ingalls, and James J Abbas. Transforming clinical rehabilitation into interactive multimedia. In *Proceedings of the 19th ACM international conference on Multimedia*, pages 937–940. ACM, 2011.
- [11] Ioannis Theoklitos Paraskevopoulos, Emmanuel Tsekleves, Cathy Craig, Caroline Whyatt, and John Cosmas. Design guidelines for developing customised serious games for parkinson’s disease rehabilitation using bespoke game sensors. *Entertainment Computing*, 5(4):413–424, 2014.
- [12] Sabine C Koch and Thomas Fuchs. Embodied arts therapies. *The Arts in Psychotherapy*, 38(4):276–280, 2011.

- [13] Citlali López-Ortiz, Deborah J Gaebler-Spira, Sara N McKeeman, Reika N McNish, and Dido Green. Dance and rehabilitation in cerebral palsy: a systematic search and review. *Developmental Medicine & Child Neurology*, 61(4):393–398, 2019.