When Competitors Share Data: An Analysis for Personalizing and Non-Personalizing Firms

We analyze a case where two competing firms, a personalizing firm that makes product recommendations and a non-personalizing firm that does not recommend products, compete with each other and may participate in a data sharing alliance (in which the non-personalizing firm shares its data with the personalizing firm). The customer is strategic as she first visits the personalizing firm to find out what product to purchase among the recommended ones and may purchase the product eventually from either of the firm. The customer distributes all her purchases across both the firms - she purchases from the personalizing firm to maintain a profile quality with the firm (to continue to obtain good recommendations) and from the non-personalizing firm to benefit from the lower prices of that firm. We analyze this setup in a two stage game to find the subgame perfect Nash Equilibrium where in the first stage firms decide to share data, or not, and in the second stage they play a simultaneous price game given the strategic customer. When the personalizing firm uses shared data, the rate of improvement of profile increases through (i) an increased learning rate (learning rate influence) and (ii) an increased knowledge about customers in general (profile influence). We find that the non-personalizing firm is always willing to share its data with the personalizing firm, but the personalizing firm uses it only when the learning rate influence dominates the profile influence. When the personalizing firm improves its system and both firms share data, its own profit always increases and under certain conditions the profit of the non-personalizing firm also increases. Finally, social surplus may increase as well when firms share data. This result has important implications for the policy-makers controlling information sharing between firms.

Key words: Recommender systems, Duopoly, Simultaneous more price game, Data sharing, Nash-Equilibrium

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1 Introduction

The increased volumes of e-commerce has generated vast amounts of data about customers, which can be used to retain old markets as well as create new ones. These are achieved by investing in technologies that analyze and use data for value added services. One such service is providing product recommendations. Examples of companies that provide such services are Amazon.com, Target, Costco, Home Depot, etc. – we hereafter refer to them as *personalizing firms*.

Recommender systems gather knowledge about the preferences of the customers for recommending products through data collected about them, which includes demographic and psychographic information and past online transactions (Cao and Li 2007, Konstan and Riedl 2012, Schafer et al. 2001). The predictive knowledge gleaned about a customer's preferences is referred to as the *profile*[1]. It is known that customers value the recommendation services provided by the personalizing firm, which increases its sales (Pathak et al. 2010). Since recommender systems use profiles of customers to recommend products, firms should invest in efforts to create better profiles.

One of the important concerns of Operations Management is improving the efficiency of a system, which affect the profile qualities of customers. Altering and improving the algorithms that power these systems is certainly one way of improving the profile quality. But that is not the only way. Recent works in the area of recommender systems suggest that profile quality improvements can be archived by using additional data; e.g., Umyarov and Tuzhilin (2009) demonstrate improvements in recommendations by incorporating data from multiple sources; similar discussions also appear in Chen et al. (2010). Additional data contains various useful information such as distributions of items purchased (common between both the firms), frequencies of the customer visits and transactions, etc. These information may help the recommender system improve its *learning rate* about the preferences of customers by reducing the uncertainties that unusually negatively affect the recommender system algorithm's accuracy. Also, additional data may provide better estimates about the customer preferences in general which would improve profiles of all the users. For the personalizing firm, the best source of data could be the competitor who sells similar products to

[1]The profile, for instance, is the knowledge derived by processing various known facts, previous transactions, and information about the preferences of a customer to estimate the unknown characteristics about the customer, which can be used to predict the products and services that the customer may want to receive in the future.

the same or similar set of customers. We analyze the case when the personalizing firm can obtain the data of a competing firm by making a *data sharing* alliance.

Firms sharing data with other firms including competitors is not uncommon among retailers; such activities have been recorded and studied in the practitioner and academic literature. For example, a white paper released by Grocery Manufacturers Association (GMA) states that most U.S. grocery manufacturers share data with their suppliers (Garry 2009). Likewise, in the direct marketing industry, more than 600 catalog marketers exchange information about purchases of individual customers with competitors (Jackson 1996, Mathur et al. 2007). For instance, Abacus B2B alliance is a data sharing arrangement between 352 co-op retailers who share 75 million names and 1.2 billion transactions. There are many such alliances and in all of them, retailers who compete share customer level data (Multichannel Merchant 2017). Many experts believe that the future of retailing is moving towards a direction where competing retailers will increasingly share data with each other (Yamsey 2016).

We particulary focus on a market where the personalizing firm competes with another firm that does not provide recommendations – hereafter referred to as a *non-personalizing firm*. Not every firm possesses the capital to invest in a recommender system because these systems are very expensive. In these lines, Google recently warned that systems relying on machine learning (such as recommender systems) have massive costs associated with them (Lipton 2015). Hence, many less prominent electronic retailers still sell products without providing recommendation services. For example, the online bookseller *buybooksontheweb.com* does not provide recommendation services to its customers. Similarly, while iTunes provides music recommendations (through their toolbar "Genius" for music items, another online firm *mp3million.com* sells music without doing so.

We consider customers who are strategic regarding the process of purchasing products. They purchase products from both the firms after evaluating several important factors, that we illustrate in the following example. Suppose the customer wants to purchase a song of Phil Collins. She visits iTunes and searches some offerings of songs by Phil Collins that iTunes recommends to her (the search results are generally personalized based on the preferences of the customer), and suppose after reviewing the offerings she purchases "Son of Man" from iTunes. This song which she finds worthy of purchasing is called her *preferred product*. Next time, the customer may search another song of the same singer at iTunes and suppose she likes "Take me Home" and considers it to purchase, which is now her preferred product. However, this time the customer purchases that song from *www.mp3million.com* where the song is available for a lower price than iTunes. In this manner, the customer distributes her purchases between the two firms. We consider a continuous time model where fractions of all the purchases are distributed between the two firm. Bank (1999) reports such price-shopping behavior in the retailing context where customers obtain recommendations from Amazon.com, but use these recommendations to purchase similar products from other firms. Typically prices at non-personalizing firms are lower – Moon et al. (2008) and Pathak et al. (2010) empirically find that personalizing firms charge a premium for the recommendation services they provide. These finding are largely consistent with the earlier research by Smith and Brynjolfsson (2001) who find that the firms that provide differentiated services (which would expectedly include recommendation services) often charge a price premium (in our model, price of the personalizing firm is allowed to be higher or lower than that of the non-personalizing firm)

Purchasing a (possibly) cheaper substitute from the non-personalizing firm may increase the customer's surplus. However, this results into personalizing firm missing the information about the preferences of the customer. Consequently, an opportunity to improve the customer's profile is lost. For example, in our illustration, when the customer purchases "Son of Man" from iTunes, iTunes registers that song as something that the customer likes, since, expectedly, a purchase is a strong indicator of the tastes of the customer. iTunes uses this knowledge about the customer's transaction to improve the profile of the customer, which reduces her fit cost (average cost incurred for not always buying her ideal product (Beach 1993)). In future searches the customer may get recommendations with lower fit costs, which should increase the surplus of the customer. However, when the customer purchases the song "Take me Home" from *mp3million.com* next time, iTunes does not register this purchase (and therefore, fit cost does not reduce) since the purchase happens at the competitor's store. The customer *balances the trade-off between reducing the fit cost and buying a product with a lower price* by purchasing from both firms. Note that it is possible that the customer does not find the song "Take me Home" at the non-personalizing firm, and instead, purchases a different song of the same singer, perhaps after some more search and examination of other songs (which may increase her total cost). We consider both the cases.

Now, the important question is, given this strategic behavior of the customer, how does the personalizing firm benefit from data sharing (where the non-personalizing firm shares its data with the personalizing firm and the latter uses it)? On one hand, if the profile quality of the customers are improved from additional data, the personalizing firm can increase its price (and profit) since the fit costs of the customers reduce. On the other hand, the customers then rely less on the purchases they make from the personalizing firm for maintaining their profile qualities. Thus, by using shared data, personalizing firm may decrease its own demand, and potentially its profit as well. Therefore, data sharing cannot always benefit the personalizing firm. Then, under what conditions the personalizing firm would like to participate in the data sharing alliance and use shared data? Also, when the personalizing firm improves the profiles of customers and increases their surpluses by making better recommendations, it may charge a higher price, which may decrease the surpluses of the customers. Does that necessarily decrease the social surplus? The answers of these questions are not clear without a proper analysis of a model considering the effect of data sharing on profile quality.

The setup is a game between two firms who decide about entering into data sharing alliance (or not entering) in the first period. The shared data may contain information about customers, such as demographic information, spatial information (e.g., zip code and neighborhoods), age group, ratings and reviews, coupons redeemed, and items purchased together in same transactions (we assume that to protect the privacies of customers, the shared data excludes personally identifiable information (e.g., name, phone numbers, address, SSN, etc.) All this information shed valuable insights on the general characteristics of customers and their preferences. In the second period, the two firms play a simultaneous move price game. The customers purchase products of the firms in the second period. We derive a subgame-perfect Nash equilibrium for the firms and the customers.

A customer optimally distributes her purchases across the two firms to maximize her surplus, rather than purchasing exclusively at one of the firms. An important question that may be raised at this point is that, are customers strategic to forego the cheaper products sometimes in order to purchase the products of the personalizing firm to build profiles and take advantage of the recommender system? Various articles in empirical research and practitioner literature indicate that customers do behave strategically. Pathak et al. (2010) provide statistically significant evidence that recommendation quality is positively correlated with the price premium charged by the personalizing firm. They note that a personalizing firm accumulates increasing amount of information about the customers to make better recommendations in the future, which makes using other e-commerce firms difficult for customers due to the difficulty in transferring this knowledge (profile) (i.e., customers are more likely to pay the premium with improved profile quality). Moon et al. (2008) also provide statistically significant evidence about the correlation between price premium and personalization service. Therefore, clearly, not only the customers have embraced the fact that the personalizing firms charge premiums along with the recommendation services, they also realize the benefits of building profile with the personalizing firm, and its relationship with the recommendations they obtain.

Further, a study conducted by a marketing research firm confirms that the customers indeed forego options at other retailers and purchase products from Amazon so that Amazon can learn their preferences and make better future recommendations (Yu 2012). Besides, Ozmen (2005) argues at a higher level that recommendations are an add-on to the products customers purchase, and the premium is for that additional service. This observation is in line with the past claims that the customers are often willing to pay a premium to avail quality services (Smith and Brynjolfsson 2001). In addition, the elite personalizing firms, such as Amazon, often educate customers about the advantages of creating profiles by being loyal to the firm (Amazon 2017). Along these lines, some academic studies have also noted that the customers are aware of this advantage (Kramer et al. 2007).

We find many insights that are of significance to the operations of recommender systems. Interestingly, we find that the non-personalizing firm always wants to participate in data sharing. It is the personalizing firm that wants to use the shared data only under certain conditions. This is an important result because generally the expectation would be that the improved profile quality should increase the profit of the personalizing firm by letting it increase its price, as implied by the past literature. We find that the personalizing firm participates in data sharing and uses the shared data only when the influence of data sharing on improving the learning rate of the recommender system is more than a threshold. The reason for this limited region where the personalizing firm is willing to participate is that data sharing reduces the reliance of the customers on transactions with the personalizing firm to improve their profiles, which decreases its demand (and the demand of the non-personalizing firm subsequently increases). The personalizing firm participates in data sharing when this decrease in demand, which it balances with an increased price, does not hurt its profit. For the non-personalizing firm, data sharing is a unique opportunity to monetize from its data, which it collects anyway but that does not provide any direct contribution to its bottom line. Without investing in any advanced technologies, the non-personalizing firm can benefit simply by providing its data to the personalizing firm. Even when the surplus of the customers decrease, social surplus may still increase when data is shared.

We also find that when the personalizing firm improves its recommender system accuracy (e.g., by implementing a new algorithm for recommendations), the profit of the personalizing firm always increases (whether data sharing happens or not). However, the profit of the non-personalizing firm may decrease under certain conditions when the personalizing firm is using the shared data. This happens because the customers start purchasing more from the personalizing firm to improve their profile qualities faster (through better recommender system and the use of shared data by the personalizing firm), leading to a simultaneous decease in the demand of the non-personalizing firm. Even though the non-personalizing firm increases its price to extract some of the surpluses of the customers from the improved profile qualities, it may not be sufficient sometimes to increase its profit. When the data is shared and the effect of shared data on the learning rate improves, both firms have increased profit. Thus, the non-personalizing firm free-rides on the improved learning rate of the personalizing firm, which also means that the non-personalizing firm has an incentive to adjust its internal procedures of collecting and storing data so that when the data is shared, the learning rate improvement is as high as possible.

Results of this research may also influence policies regulating data sharing and information dissemination across competitors. Critics often argue that firms collecting data about customers have potential to reduce customer surplus because firms can exploit the customers (Newman 2017). Data sharing is another way of obtaining more information about customers (although we assume that privately identifiable information are not shared, using data mining techniques, firms can infer preferences of their customers by finding similar customers based on similarities of transactions). However, recently, experts are arguing that responsible data collection by firms and proper use of data for generating better services can benefit customers, and customers are now realizing that. Further, customers are less concerned than before about data collection by firms provided they are able to get better services (Thierer 2013, Kelly 2017). As Morey et al. (2015) find from an extensive survey of consumers, “Our surveys reveal that when data is used to improve a product or service, consumers generally feel the enhancement itself is a fair trade for their data.” Our research should help in this line of reasoning as we show that firms can generate substantial social surplus when data is properly used by companies for providing value added services.

2 Literature Review

Recommender systems have gained increasing attention in the operations management literature because of their direct and indirect impact on sales, volume, and inventory. Recent research has focused on making recommendations by considering the product inventory, future demands and returns (Demirezen and Kumar 2017). Besbes et al. (2016) propose a model in the context of a content management website for predicting the series of links that a reader may click. Fleder and Hosanagar (2009) found that recommender systems may lead to consumption of an increased variety of products by the consumers, however, these consumers increasingly see the same types of products. Later, Hosanagar et al. (2014) show that when recommender systems increase the variety of products consumed by the customers, it does so through an increase the volume of sales, and for a given volume consumers increasingly purchase similar products. Jiang et al. (2017) has proposed a new recommendation method to recommend products that are both relevant and diverse. Netessine et al. (2006) propose a method for cross-selling a package of complementary products when a customer is searching a specific product of her interest. They analyze two issues: (1) how to select packaging complements, and (2) how to price product packages to maximize profits. To solve the problem they propose a stochastic dynamic program. Our research contributes in this line of research by considering another important aspect of recommender systems - how should firms decide whether or not to create a data sharing alliance by considering the effect of purchases by customers and data sharing on profile quality.

There are several other areas of literature that are connected to our research to various extents, such as personalization/recommender system, customized products, loyalty programs, and information/data sharing. Here, we also compare and contrast our work with past literature in order to highlight our contributions. The majority of the research in the area of recommender systems have focused on designing and improving them (e.g., see Garfinkel et al. 2006, 2008). An extensive review of recommendations methodologies is provided by Breese et al. (1998) and Adomavicius and Tuzhilin (2002). Bodapati (2008) examines the influence of recommender system on the purchase behavior of customers. Senecal et al. (2005) compare the search and navigation patterns of customers who consider product recommendations with those who do not when they browse through websites of personalizing and non-personalizing firms. None of these studies consider the economic analysis of profile quality and data sharing, which is the focus of our work.

Akin to recommendations, customized products also better meet the preferences of a customer. Dewan et al. (2003) study competition between two customizing firms to derive equilibrium prices. Syam et al. (2005) investigate when to customize in duopoly. Syam and Kumar (2006) also investigate a duopoly, and find that the firms should offer both standard and customized products to maximize profit. In none of these studies, customization benefits depend on a customer's past purchases at a customizing firm. In addition, unlike recommendations, customers do not have a way to transfer customization benefits across firms.

Loyalty programs, like recommendation, provide immediate value to the customer, and can grow with increased patronage. In this context, Biyalogorsky et al. (2001) examine how a firm should strike a balance between loyalty rewards and attractive prices to maximize profit. Lewis (2004) provides a framework to measure the influence of loyalty reward programs on consumer retention, while Meyer-Waarden (2008) study how loyalty programs induce customers to continue purchasing from the firm. Unlike rewards, however, recommendations are transferable, i.e., the customer can use the recommendations provided by a personalizing firm to find similar products at another firm.

Research in the area of supply chain, economics, and marketing has recognized the importance of information sharing between multiple parties. Stefansson (2002) recognizes the importance of data sharing between manufacturers and suppliers and identifies various means of information sharing channels between the entities. Through case studies, he shows that many small and medium sized firms are left out in this important activity because they do not have the capital to implement Electronic Data Exchanges. However, the internet has the potential to change this scenario. Tengberg (2013) provides an extensive review on the motivations that drive retailers to share data. These studies, unlike ours, do not study the strategic interactions between firms where they behave like rational players, which choose to share or not share data based on their profit functions.

Several research are worth citing since they study the roles of uncertainties in demand and costs in information sharing. Gal-or (1985) analyzes a setup where two symmetric firms (e.g., in their technical abilities) have similar assessments about the uncertainties in the environment affecting demand from customers. Information sharing reduces the correlation in outputs generated after estimating the demand. She concludes that information sharing does not happen in the equilibrium because if any sharing happens at all, firms share incomplete information in equilibrium and in that case losses outweigh the gains. Li (1985) analyzes a specific case where in an oligopoly firms may want to share information about demand uncertainties or private costs. He concludes that in the equilibrium firms do not share the information about demand but share the information about costs. Gal-or et al. (2008) show that in a market with one manufacturer and two retailers, the manufacturer may choose to share information with the retailer functioning in a less competitive environment. This is because in that kind of environment, the channel efficiencies are generally higher and therefore, benefits accrued are higher if the inefficiencies are reduced or eliminated by sharing information. Finally, Raith (1996) has two key findings related to when firms reveal profit functions: (1) Letting the rivals acquire a better knowledge of their respective profit functions leads to a higher correlation of strategies, the profitability of which is determined by the slope of the reaction curves. (2) Letting the rivals acquire a better knowledge of one's own profit function is always profitable. Eventually, these analyses reveal that information sharing is profitable for firms under certain conditions. Our results are consistent with this general finding. However, there is a major difference between data sharing as we consider and information sharing these researchers consider. These papers model the sharing of private information about profit functions, demands, and private costs. In our context, profit functions and demands are known to everyone. We do not consider costs of making recommendations in the base model, and in the extension when we consider it, again the functions are known to both firms. Our model does not rely on revealing or sharing any private information. We show that when data sharing changes the profile quality, and consequently the demand, which is known to both the parties, under what conditions the firms participate in data sharing.

3 Model Setup

We start with discussing the sequence in which the events unfold, and then we discuss the customer's and the firms' problems.

3.1 Model Context

There are two firms – a personalizing firm which makes recommendations, and a non-personalizing firm that does not make recommendations – and customers who purchase products from both firms. The sequence of events are shown in Figure 1.

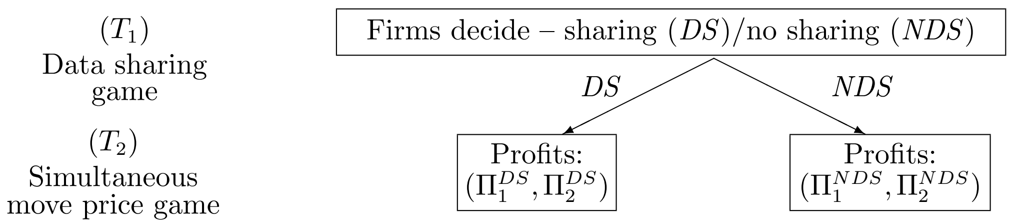


Figure 1 Game: Time epochs for the events and profits

At time T_1, the firms decide whether to share data or not. The two firms make decisions based on their expected profits (at T_2) after data sharing happens. We assume that firms would have shared data in the past and are aware of the values of the parameters that affect the profile quality after data is shared. At time T_2, the firms play a simultaneous move price game. If they decide to share data, the profit of the personalizing firm is denoted by Π_1^{DS} and the profit of the non-personalizing firm is Π_2^{DS}. If the firms do not share data, the profit of the personalizing firm is denoted by Π_1^{NDS}, and the profit of the non-personalizing firm is denoted Π_2^{NDS}. The difference in profits arise in time T_2 because the prices and demands of products at that second period are decided based on the outcome of the decisions of the first period (sharing/no sharing).

In either case (data shared or not shared), customers observe the prices and estimate their surpluses from the products these firms offer. They divide their purchases of products across the two firms to obtain maximum surpluses. The customers purchase u (where $0 \leq u \leq 1$) fraction of products from the personalizing firm and $(1-u)$ fraction of products from the non-personalizing firm. They are interpreted as purchase rates – at any instant of time, u fraction of all products purchased from the personalizing firm, and the rest from the competitor. The rates stay the same at every time instant. Customers choose rates that maximize their surpluses by deciding optimal u.

The rate of profile quality of a customer is positively affected by u, and the profile quality improves as the customer spends more time transacting with the personalizing firm at the rate u. The shared data improves the profile quality faster with an increase in time as compared to when the shared data is not used.

The products purchased by the customer (from either firm) are assumed to belong to the same category with fixed prices (calculated under equilibrium) charged by each firm. Prominent examples for products that are repeatedly purchased by customers for the similar price are music items and movies, which have been considered by previous researchers as well for a similar context (Wattal et.al. 2009). While the prices within a product category may vary slightly, we assume that the variation within a product category is not substantial and the single price approximation is reasonable, e.g., almost all songs in iTunes are sold at \$0.99.

3.2 Customer's Problem

All customers are assumed to have the same reservation price R, which is sufficiently large so that they purchase products (this assumption is similar to the one used in Tirole (1988, page 279), and we relax this assumption in Section 6.4). For now, we consider a market where most customers have purchased from the two firms for a substantially long time (we relax this assumption later in Section 6.3 to allow customers to be heterogenous in profile quality and show that there is no impact on our subsequent analyses). Suppose such customers have a profile quality x. A customer considers the cost she incurs due to typically not purchasing her ideal product, which is referred to as fit cost. The fit cost decreases if profile quality (x) increases. Thus, we assume a decreasing function in x as $(A - \xi x)$ to represent the fit cost, where A is the maximum fit cost (constraints on A are provided later). The total cost incurred by the customer when purchasing from the personalizing firm is $(A - \xi x + p_1)$. In this base setup, we consider that the customer purchases the same product from the non-personalizing firm that she finds as the preferred product at the personalizing firm's website (we will later show that the results remain the same when that is not the case, and the customer incurs an additional fit cost). Therefore, the total cost incurred by the customer from the non-personalizing firm is $(A - \xi x + p_2)$.

At an instant, the objective of the customer is to maximize surplus ($R-$ total costs) by choosing u, where the costs depend upon fractions of purchases from the two firms (u and $1-u$), the profile quality (x), and the prices of products (p_1 and p_2).

$$\max_{u} \left(R - (A - \xi x + p_1)\, u - (1-u)\,(A - \xi x + p_2)\right). \tag{1}$$

Profile quality of a customer x is a dynamic entity which improves with time (as in due course the customer transacts more with the personalizing firm). A customer who has been purchasing products only for a month will have a lower profile quality than another customer who has spent a year for the same rate of purchase u from the personalizing firm. Thus, profile quality is a function of time t. To incorporate this time dependence of the profile quality, we model the rate of profile improvement. At any point of time, the profile quality increases when a better recommender system algorithm is employed for profile creation. The effectiveness of the system (the algorithm) to process the data and generate profiles is denoted by α. A higher purchase rate u should improve the profile quality[2]; however, Harper et al. (2005) experimentally show that the recommendation quality improves at a decreasing rate with increasing customer transactions. Thus, we model the factors that positively influence the profile quality improvement as $\alpha(Bu - u^2) = \alpha(B-u)u$. $\alpha\,(B-u)$ is called the *learning rate* of the recommender engine, which interacts with the amount of transactions from the personalizing firm (i.e., u). The parameter B controls the rate at which $\dot{x}$ depends on u (in a concave fashion), and is referred to as profile augmentation constant since a higher value of

[2]Our specification of profile quality asserts that a customer's purchases are a principal determinant of the profile quality. For example, in Netflix, recommendations mainly depend on a customer's consumption history and ratings (if any), rather than secondary factors such as the customer's search history. Other factors, such as search history, vary over time, and carry less information (relative to actual purchases) about her preferences. For instance, a customer may search for a specific product but not purchase it for a variety of reasons (Alam et al. 2013, Nasraoui and Saka 2007, Suryavanshi et al. 2005).

B increases the rate of improvement of profile by increasing the learning rate for a given u. For a given B, the $B-u$ part of the learning rate controls the change in the rate with the change in u. Thus, when $u=0$, the learning rate is αB, which is higher than the learning rate at $u=1$ (learning rate is $\alpha(B-1)$). To ensure concavity, we assume $B>2$ for $0\le u\le 1$ (basically, values of $B<2$ are not relevant in this context given the experimental findings of Harper et al.). Also, we assume $A>\xi\alpha(B-1)$ so that $A-\xi x>0$.

Other factors may reduce the rate of profile improvement. One such factor is the changes in the preferences of the customer. For example, if the customer graduates from college, her music preferences may get influenced by the tastes of her new peers who may have mature tastes, different from college going students. A parameter β, called the forgetting rate, represents such changed preferences. If this rate is high, any knowledge gathered today about the customer's preferences get obsolete quickly, thereby reducing the rate of improvement of profile quality. Finally, the more the system learns about the customer and is certain about the preferences, the less remains to learn further. Thus, $\dot{x}$ decreases with an increase in x. In summary, we have $\dot{x}$ as,

$$\dot{x}=\alpha\,(B-u)\,u-\beta x. \tag{2}$$

3.2.1 Impact of data sharing on profile quality improvement Following the literature in recommender system design, we model the two-part effect of additional data on profile quality improvement. First, as Chen et al. (2010) and Umyarov and Tuzhilin (2009) suggest, the system obtains more information for computing the estimates of ratings, preferences, etc., from additional data, effectively changing the learning rate to process the transactions of the customer (u). The additional purchase transactions obtained from the non-personalizing firm could provide patterns (e.g., items purchased in the same transactions by a large number of customers) that were not observed in the transactions possessed by the personalizing firm. From these new patterns, the recommender system algorithm can make predictions that are potentially different from those which it could make without these information. Thus, the change in learning rate is proportional to $(1-u)$. Suppose η is the effectiveness of processing the additional data (naturally it has to be less than the effectiveness of processing the firm's own data, i.e., $|\alpha| > |\eta|$). The change in the learning rate is, therefore, $\eta(1-u)$. Thus, the new learning rate is $\alpha(B-u)+\eta(1-u)$. The new learning rate is consistent with the observations of Harper et al. (2005). At $u=0$, the learning rate is $\alpha B+\eta$, which is higher than the learning rate when $u=1$ (learning rate is $\alpha(B-1)$).

The second effect of data sharing, albeit more subtle, is as follows. According to Chen et al. (2010), a study of data from an online recommender system for movie ratings (MovieLens), any additional data is like a "public good" because additional data about each customer improves the predictions for all others. Essentially, the additional data (e.g., ratings for a movie ratings database added by one person helps in improving everyone's profile) provides aggregate information that allows the personalizing firm to make better predictions for the preferences of the population as a whole. This "across the board" improvement is modeled as $\delta(1-u)$, where δ is the expected general improvement and is weighted by the amount of information obtained from the shared data $(1-u)$.

Overall, data sharing influences the profile quality improvement in two ways – it provides information about the population (as if, without even knowing about a customer's u, some predictions can be made about the customer using the aggregate information, the $\delta(1-u)$ part), and, it changes the learning rate for the profile targeted to a specific customer, the $\eta(1-u)$ part. The effects of η and δ may be stronger or weaker with respect to each other; the literature does not provide any guidance in this regard. Hence, the profile quality improvement can be represented as follows when shared data is used by the personalizing firm,

$$\dot{x} = (\alpha(B-u)+\eta(1-u))\,u+\delta(1-u)-\beta x, \tag{3}$$

where $\eta,\delta>0$. Trivially, when $u=1$, effect of data sharing disappears in the expression of $\dot{x}$. Also, on setting $\eta=\delta=0$ in Equation (3), we get back Equation (2), the expression of $\dot{x}$ when no data is shared. The overall effect of data sharing on improving profile (i.e., improvement due to $(1-u)$) is not expected to exceed the effect of transactions with the personalizing firm. This serves as a boundary condition (others will be mentioned subsequently) for our solution space, which is:

$\alpha B - \delta - 2\alpha u + \eta(1-2u) > 0$. This condition is valid for every u. As the left hand side is increasing in u, we can simplify it by setting $u = 1$. Solving this provides a boundary condition that restricts the values of the constant B considered relevant in our context:

$$B > 2 + \frac{\eta + \delta}{\alpha}. \tag{4}$$

Although Equation (3) shows an intricate underlying relationship of $\dot{x}$ and the parameters (α, η, δ, u, and x), the customers only need to know the $\dot{x}$ as a function of u and x, both of which customers observe. Basically, a customer estimates the values of C_1, C_2, C_3, and C_4 in the relationship $\dot{x} = C_1 u + C_2 u^2 + C_3 x + C_4$ from experiences with the firms through past transactions. When data sharing does not happen, she estimates the values of these parameters equivalent to αB, $-\alpha$, $-\beta$, and 0, respectively. When data sharing happens, she estimates these values to be equivalent to $\alpha B + \eta - \delta$, $-\alpha - \eta$, $-\beta$, and δ, respectively. Substituting the values of C_1, C_2, C_3, and C_4 provides the respective equations of $\dot{x}$.

Now, using Equations (1) and (3), we present the surplus maximization problem of the customer for a time t as

$$\begin{aligned} &\max_u \left(R - u\left(A - \xi x + p_1\right) - (1-u)\left(A - \xi x + p_2\right)\right), \\ &0 \le u \le 1. \end{aligned} \tag{5}$$

where $x(t)$ is obtained by solving Equation (3).

3.3 The Firm's Problem

The firm's problem is modeled in two parts representing the two decisions they make. First we model the price game and then we model the data sharing game.

3.3.1 Price game setup This game takes place at time T_2. The firms play a simultaneous move price game where they maximize their equilibrium profits at an instant t. We assume that the cost of implementing the recommender system is sunk, and the marginal cost of individual recommendations is negligibly small, assumed to be 0. Similar assumptions have been made by past researchers in the context of a game involving personalizing firms (Wattal et.al. 2009) (we relax this assumption in Section 6). The profits of the personalizing and non-personalizing firms, respectively, are:

$$\Pi_1^j = up_1 \text{ and } \Pi_2^j = (1-u)p_2 \tag{6}$$

where $j \in \{DS, NDS\}$ referring to Figure 1. Prices and u depend upon the sharing decisions.

3.3.2 Data sharing game setup This event takes place at time T_1. Both firms decide to share (or not share) data simultaneously anticipating the equilibrium of the price game and the subsequent purchase decisions of customers. The firms compute their expected profits from sharing and compare them with profits from the case of no data sharing. The firms share data when the expected profits from sharing are higher.

We introduce two variables now that correspond to the solution of the equilibrium, namely k_1 and k_2, such that if personalizing firm participates in data sharing (respectively, does not participate in data sharing) then $k_1 = 1$ (respectively, $k_1 = 0$). Likewise, if non-personalizing firm shares data (respectively, does not share data) then $k_2 = 1$ (respectively, $k_2 = 0$). The variable $k_1 = 1$ means the personalizing firm is willing to receive and use the shared data. The variable $k_2 = 1$ means the non-personalizing firm is willing to transfer its data to the personalizing firm. Using these variables, we can now redefine the (data sharing) variables η and δ in terms of exogenous parameters as $\eta = k_1k_2\psi_1$, and $\delta = k_1k_2\sigma_1$. The (exogenous) parameters are defined as follows: ψ_1 is the *learning rate influence*, and σ_1 is the *profile influence*. Table EC.1 provides the notation and summaries of all the variables needed in the model.

4 Price Game Equilibrium and Analysis

We sequentially solve the model to derive the sub-game perfect Nash Equilibrium, starting from the customer's problem. First, we show the solution for the customer's problem who maximizes her surplus. At a time t, $x(t) = (1 - e^{-\beta t})\frac{Bu\alpha + \delta - u(\delta - \eta + u(\alpha+\eta))}{\beta}$. Assuming $t \to \infty$ for customers who have been purchasing from the two firms for a long duration (our current focus) we get,

$$x = \frac{Bu\alpha + \delta - u(\delta - \eta + u(\alpha+\eta))}{\beta} \tag{7}$$

Next, we show the expressions of profits, prices, and fraction of purchases for when data sharing happens and when it does not happen, for given prices of the two firms.

LEMMA 1. *The optimal fraction of purchases from the personalizing firm ($0 \leq u \leq 1$) when*

(i) Data sharing happens ($k_1 k_2 = 1$):

$$u = \begin{cases} \frac{-\beta p_1 + \beta p_2 + \xi(B\alpha - \sigma_1 + \psi_1)}{2(\alpha+\psi_1)\xi}, & \textit{if } \max\left\{2, \frac{\beta p_1 - \beta p_2 + \xi(\sigma_1 - \psi_1)}{\xi\alpha}\right\} < B < \left(2 + \frac{\xi(\psi_1+\sigma_1)+\beta p_1 - \beta p_2}{\xi\alpha}\right) \\ 0 \textit{ or } 1 & \textit{otherwise.} \end{cases} \tag{8}$$

(ii) Data sharing does not happen:

$$u = \begin{cases} \frac{-\beta p_1 + \beta p_2 + \xi B\alpha}{2\alpha\xi}, & \textit{if } \max\left\{2, \frac{\beta p_1 - \beta p_2}{\xi\alpha}\right\} < B < 2 + \frac{\beta p_1 - \beta p_2}{\xi\alpha} \\ 0 \textit{ or } 1, & \textit{otherwise.} \end{cases} \tag{9}$$

The proofs are provided in the E-Companion. To ensure a duopoly, we impose the condition $0 < u < 1$, which translates into the conditions shown in the Lemma. If the conditions on B are not satisfied in the respective cases, one of the firms covers the entire market. That is not a case of interest in our context. At this point, the lower and higher limits of B are in terms of variables considered exogenous at this stage. These are additional constraints that must be considered along with Equation (4) to obtain the feasible space of parameter values so that u is relevant in our problem context. In other words, values of B that do not follow these conditions generate markets that are uninteresting or impractical. Altogether, the condition on B can be written as the following:

$$\begin{aligned} &\max\left\{2, \frac{\beta p_1 - \beta p_2}{\xi\alpha}, 2 + \frac{\psi_1 + \sigma_1}{\alpha}, \frac{\beta p_1 - \beta p_2 + \xi\sigma_1 - \xi\psi_1}{\xi\alpha}\right\} \\ &< B < \min\left\{2 + \frac{\beta p_1 - \beta p_2}{\xi\alpha}, \left(2 + \frac{\xi\psi_1 + \xi\sigma_1 + \beta p_1 - \beta p_2}{\xi\alpha}\right)\right\}. \end{aligned}$$

4.1 Equilibrium

Since u and $(1-u)$ are functions of prices (p_1 and p_2), we next focus on determining the equilibrium prices. To determine the Nash equilibrium prices and profits, firms solve the profit maximization games in time T_2 for each case. Following Lemma shows the results of different cases. The boundary conditions, derived by substituting the equilibrium expressions in conditions specified in Lemma 1, ensure that the markets have duopoly.

Lemma 2. *Following are the Nash Equilibrium results when*

1. *data sharing happens* ($k_1k_2=1$)*:*

   (i) $u^* = \frac{(2+B)\alpha-\sigma_1+3\psi_1}{6(\alpha+\psi_1)}$.

   (ii) The price and profit, respectively, of the personalizing firm are $p_1^* = \frac{\xi((2+B)\alpha-\sigma_1+3\psi_1)}{3\beta}$ *and* $\Pi_1^* = \Pi_1^{DS^*} = \frac{\xi((2+B)\alpha-\sigma_1+3\psi_1)^2}{18\beta(\alpha+\psi_1)}$.

   (iii) The price and profit, respectively, of the non-personalizing firm are $p_2^* = \frac{\xi(4\alpha-B\alpha+\sigma_1+3\psi_1)}{3\beta}$ *and* $\Pi_2^* = \Pi_2^{DS^*} = \frac{\xi((4-B)\alpha+\sigma_1+3\psi_1)^2}{18\beta(\alpha+\psi_1)}$.

   A market with duopoly exists when $\max\left\{\left(\frac{\sigma_1-3\psi_1}{\alpha}-2\right),\left(2+\frac{\psi_1+\sigma_1}{\alpha}\right)\right\} < B < 4$.

2. *data sharing does not happen* ($k_1k_2=0$)*:*

   (i) $u^* = \frac{2+B}{6}$.

   (ii) The price and profit, respectively, of the personalizing firm are $p_1^* = \frac{\xi(2+B)\alpha}{3\beta}$ *and* $\Pi_1^* = \Pi_1^{NDS^*} = \frac{\xi(2+B)^2\alpha}{18\beta}$.

   (iii) The price and profit, respectively, of the non-personalizing firm are $p_2^* = \frac{\xi(4-B)\alpha}{3\beta}$ *and* $\Pi_2^* = \Pi_2^{NDS^*} = \frac{\xi(4-B)^2\alpha}{18\beta}$.

   A market with duopoly exists when $2 < B < 4$.

Altogether, the constraint on B as $\max\left\{\left(\frac{\sigma_1-3\psi_1}{\alpha}-2\right),\left(2+\frac{\psi_1+\sigma_1}{\alpha}\right)\right\} < B < 4$ ensures that we stay in a parameter region that reflects the equilibrium market structure of our interest.

4.2 Analysis

Before we begin, it is instructive to compare our model with a classic model of Bertrand competition with same marginal costs and symmetric demands across firms with respect to prices. Apparently, there is no differentiation between the two firms as the customers obtain the same fit costs from both firms. Thus, both firms should charge their marginal costs (0 in this case) and share the markets equally (i.e, $u = 1-u = 0.5$) (Mas-Colell et al. 1995). However, in our context, as evident from Lemma 2, the prices are non-zero and unequal. Also, the demand of the two firms are unequal (i.e., $u \neq (1-u)$). Thus, recommendations are indeed a source of differentiation between firms, and the differentiation provided by the recommender system is not dissipated even when a customer may purchase a recommended product from the non-personalizing firm. In all cases, the price of the personalizing firm is higher than that of the non-personalizing firm in equilibrium.

4.2.1 Recommender System Effectiveness The following proposition summarizes the effect of improving the recommender system effectiveness (α).

PROPOSITION 1. *A. When firms do not share data and α increases,*

(i) The customer purchases from the two firms at the same rates and her profile quality improves.

(ii) The prices and profits of both firms increase.

B. When non-personalizing firm shares data and α increases,

(i) The customer purchases more from the personalizing firm and her profile quality improves.

(ii) The profit and price of the personalizing firm increase.

(iii) The price of the non-personalizing firm always increases, but its profit increases iff $\sigma_1 < (4-B)(\alpha+\psi_1)-(B-1)\psi_1$.

When no data sharing happens, the customer does not change her purchase rate from either firm in equilibrium. However, improved recommender system effectiveness improves the customer's profile, leading to a reduced fit cost. Thus, both firms increase prices to extract some the surplus of the customer. Increased prices cause increased profits of both the firms.

When the non-personalizing firm shares its data with the personalizing firm, the customer is positively affected by both the recommender system effectiveness and the data sharing alliance. Hence, as the recommender system improves, the customer increases her purchase from the personalizing firm in order to compound the positive impact on profile quality. This is consistent with finding of the past literature – Hosanagar et al. (2014) find that recommendations increases the sales volume, which would imply that improved recommendations will increase sales volume. As a result, the profile quality improves (see the proof of Proposition 1). Thus, both firms increase their prices to extract a part of the increased surplus for the customer (via her reduced fit cost). This increased price increases the profit of the personalizing firm. For the non-personalizing firm, the profit increases only when the condition in 1.B.(iii) is satisfied. This condition is not satisfied for high values of B; when B is very close to 4, the first term in the right hand side approaches 0. Since $\sigma_1, \psi_1 > 0$, the condition is not satisfied. Intuitively, a high profile attenuation constant (B) helps in fast increase in the profile quality. In that case, the customer wants to reduce its fit cost primarily by increasing her purchases from the personalizing firm significantly, as is evident from the first order derivative $\frac{du^*}{d\alpha} = \frac{\sigma_1+(B-1)\psi_1}{6(\alpha+\psi_1)^2}$ which attains higher values when B is large. It also, implies that at higher B the demand of products of the non-personalizing firm decreases drastically with an increase in α. Increase in price does not then compensate the lost revenue from the decrease in demand, causing its profit to decrease. Otherwise, it free-rides on the improved recommender system of the personalizing firm when the condition is satisfied.

The point to note here is that when α increases, u^* does not change when no data is shared, and it increases, i.e., $1-u^*$ decreases, when data is shared. This may have a negative impact on the profit of the non-personalizing firm when it decides to share its data (when 1.B.(iii) is not satisfied). Although the non-personalizing firm cannot control the change in α because it depends on the factors that affect the cost of implementation of the recommender system by the personalizing firm, it may want to be vigilant about such possibilities before it decides to part with its data and transfer it to the personalizing firm. If it can foresee a potential loss in revenue in the future because of a potential increase in α after data sharing happens, it may want to not share its data. Also, this result has direct implications to the area of inventory based recommendations in the operations literature (Demirezen and Kumar 2017), since data sharing changes u^* with an increasing in α. When data is not shared and u^* remains constant with an increase in α, the personalizing firm does not need to worry about any sudden changes in the product inventory (which generally happens because of changes in the demands of products). However, when data is shared and α increases, the personalizing firm may have to adjust its recommendations to avoid sudden drops in inventory with the increase in u^*, which otherwise may cause denied requests for services (such as DVD rentals for movies not present in the inventory) and consequent customer disenchantment.

The surpluses of the customers may increase when α increases, with or without data sharing. When no data sharing happens, customer surplus increases when $B(B+16) > 44$. If the firms participate in a data sharing alliance, the customer surplus increases when $\sigma_1 < -(B-1)\psi_1 + (\alpha + \psi_1)\sqrt{(-44+B(16+B))}$. Basically, even when the firms increase prices, under these conditions, the profile quality improvements are sufficiently high so that the decrease in fit costs are more than the increase in prices, causing higher surpluses for the customers.

Next, we analyze the effects of learning rate influence and profile influence, which are valid only when both firms participate in data sharing.

4.2.2 Learning rate influence The following proposition summarizes the impact of changes in the learning rate influence (ψ_1).

PROPOSITION 2. *When ψ_1 increases, u^* decreases, the profile quality increases iff $\psi_1 > -\frac{\sqrt{5}}{3}\sigma_1 + \alpha(\frac{\sqrt{5}}{3}(B-1)-1)$, both firms charge higher prices, and their profits increase.*

An increase in ψ_1 causes an increase in the learning rate, because of which the customer can afford to purchase less from the personalizing firm. The profile quality increases when $\alpha\left(\frac{\sqrt{5}}{3}(B-1)-1\right) < 0$, i.e., when B is small. In that case, the contribution of increased ψ_1 is much higher in profile improvement than the profile augmentation constant (B). However, when B is large, it is possible that the condition provided in Proposition for the increase in profile is not satisfied and the profile quality reduces. It happens because when B is large, the adverse impact of the decreased u^* gets amplified. Since the firms get more differentiated with improved ψ_1, they increase their prices, and their profits increase. In equilibrium, intuitively the profit of the personalizing firm increases because the reduction in u^* is small, just sufficient to benefit from the lower price of the non-personalizing firm while not impacting the profile quality substantially, and the effect of decreased u^* is dominated by the increase in price. The increase in the purchase rate from the non-personalizing firm increases the firm's profit even if it does not change its price. In equilibrium, naturally then, the increase in both price and demand of the non-personalizing firm increases its profit.

4.2.3 Profile influence The following proposition summarizes the impact of changes in the profile influence (σ_1).

PROPOSITION 3. *When σ_1 increases, u^* decreases, profile quality increases if $\sigma_1 > \frac{-9}{5}\psi_1 + \alpha(B - 2.8)$, price and profit of the personalizing firm decrease, and those of the non-personalizing firm increase.*

When data is strongly compatible and the profile influence increases, the profile quality of a customer improves even when they do not purchase from the personalizing firm (see Equation (3)). Hence, a customer can reduce her purchases from the personalizing firm and still maintain her profile quality, which happens in the equilibrium. In equilibrium, profile quality always increases when $B < 2.8$. This is because a small B does not contribute significantly in the improvement of the profile quality, and therefore, the contribution of recommender system effectiveness is also small (as it happens through B). The major contribution in profile quality improvement is made by the increased profile influence. When B is large (more than 2.8), then the contribution of the learning rate is high, and a reduced u^* causes a decrease in the profile quality when the condition is not satisfied. The personalizing firm reduces its price to temper the loss of demand. Decrease in both the price and demand eventually reduces the profit of the personalizing firm.

On the other hand, the non-personalizing firm increases its price in equilibrium. Increased σ_1 helps in improving the profiles of customers across the board through the shared data. The customer starts purchasing more from the non-personalizing firm, which provides some room for the non-personalizing firm to increase its price. Because of the increase in both the price and demand, the profit of the non-personalizing firm increases.

4.3 Profile Quality of a Customer: DS vs NDS

In this section, we compare the changes in the profile qualities of customers when firms participate in data sharing as compared when they don't. Following proposition provides the conditions for the parameters when the profile quality improves after personalizing firm uses shared data.

PROPOSITION 4. *The profile quality of a customer is better than when data is not shared iff:* $\sigma_1 > \frac{1}{5}\left((-14+5B)\alpha - 9\psi_1 + \sqrt{(\alpha+\psi_1)\left((14-5B)^2\alpha + 36\psi_1\right)}\right) = \sigma_{x_{th}}$.

Surprisingly, although data sharing would improve profile quality if everything else remain the same, the changes in u^* may cause a situation when the profile quality of the customer may be lower after the firms share data. It happens when the condition in the proposition is not satisfied, i.e, when the profile influence is smaller than a threshold value ($\sigma_{x_{th}}$). As we mentioned earlier, at lower values of σ_1, the customer purchases more from the personalizing firm. However, u^* has diminishing effect on the profile quality. It turns out that if σ_1 is lower than $\sigma_{x_{th}}$, the effect of high u^* is not sufficient to have a profile quality higher than if the firms did not have shared data.

The ultimate effect of profile quality is on the fit cost, and consequently on the surplus. The important question is that whether an increase in profile quality can lead to higher customer and social surplus, or not. We numerically find that when data is shared, the surplus of a customer reduces because of the increased prices of the firms, and even increased profile quality does not provide enough surplus to cover the increased prices of both firms (the value of R is varied from 2 to 5, A is varied from 1 to 4, α is varied 0 to 1, β is varied from 0.1 to 1.5, B is varied from 2 to 4). However, the social surplus (profit of firms + customer surplus) may still increase (e.g., when $B = 2.85$, $\alpha = 0.2$, $\beta = 1$, $R = 5$, $A = 3$, $\psi_1 = 0.00878$, $\sigma_1 = 0.1521$, increase in social surplus is 0.01484).

5 Data Sharing Game Equilibrium And Analysis

Next, we solve the game of data sharing played at time T_1. At this stage, firms use their expected profits at time T_2 on sharing and not sharing data to decide whether to participate in a data sharing alliance.

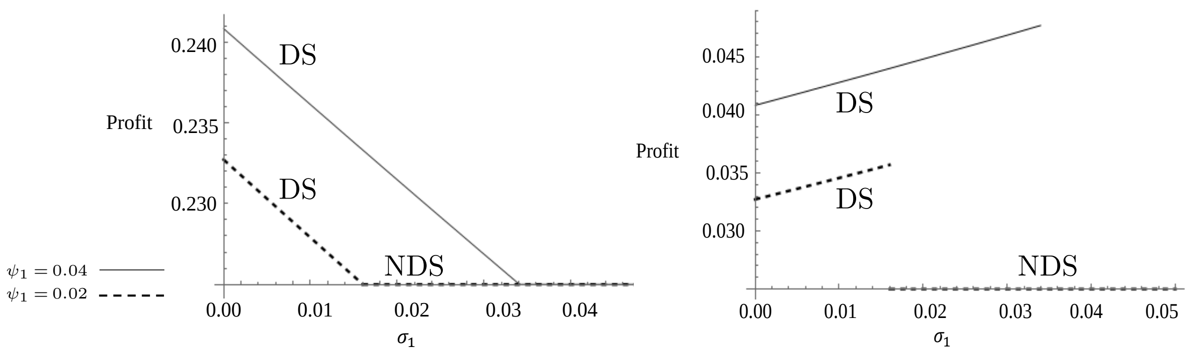
PROPOSITION 5. *Both firms willingly share data, i.e.,* $k_1 = k_2 = 1$, *when* $c_1 > 0$ *where* $c_1 := \psi_1 > \frac{1}{18}\left(6\sigma_1 + (2+B)\sqrt{\alpha}\left(-(4-B)\sqrt{\alpha} + \sqrt{(4-B)^2 + 12\sigma_1}\right)\right) = \psi_{1_{th}}$. *Otherwise,* $k_1 = 0$ *and* $k_2 = 1$, *and therefore,* $k_1 k_2 = 0$, *i.e., data sharing does not happen.*

The above proposition lays down the conditions for the pure strategy Nash-Equilibrium when firms are willing to participate in data sharing. The personalizing firm does not always make more profit by using the shared data, and the reason is that an increase in ψ_1 increases its profit, however, an increase in σ_1 decreases its profit, as discussed in Section 4.2. The condition c_1 corresponds to the parameter space where the strongly compatible data provides more profit to the personalizing firm than the profit it obtains when data is not shared. It states that the value of ψ_1 for a given σ_1 must be at least $\psi_{1_{th}}$. Interestingly, the non-personalizing firm is always ready to share its data because whether ψ_1 increases or σ_1 increases, the non-personalizing firm always obtain a higher profit.

5.1 Changes in Equilibrium Solution for DS/NDS with Data Sharing Parameters

In this section, we provide a consolidated analysis of the equilibrium to show how the decision to share the data changes with the changes in parameters, i.e., how the solution of Stage 1 of the game changes with the changes in ψ_1 and σ_1. Figure 2 illustrates the parameter space when firms switch between sharing and no-sharing regimes when $\alpha = 0.2$, $B = 2.5$, $\beta = 1$, and $\xi = 1$. In Figure 2(a), the profit of the personalizing firm decreases with an increase in σ_1 (as discussed in Proposition 3(b)). When the profit of personalizing firm by using shared data becomes smaller than the profit by not using the shared data, the data sharing does not happen. This happens in Figure 2(a) when $\sigma_1 \geq 0.02$ for $\psi_1 = 0.02$ and $\sigma_1 \geq 0.04$ for $\psi_1 = 0.04$. In these cases, the effects of ψ_1 and σ_1 disappear, and therefore the profit of the personalizing firm does not change with σ_1.

Figure 2(b) shows the sharing and no-sharing profits of the non-personalizing firm. The profit of the non-personalizing firm increases with σ_1 (as discussed in Proposition 3(b)) until $\sigma_1 < 0.02$ for $\psi_1 = 0.02$ and $\sigma_1 < 0.04$ for $\psi_1 = 0.04$. At $\sigma_1 = 0.02$ for $\psi_1 = 0.02$, and at $\sigma_1 = 0.04$ for $\psi_1 = 0.04$, the personalizing firm stops using the shared data, and therefore at this instant, profit of the non-personalizing firm reduces to its profit without data sharing (and then remains constant with an increase in σ_1).



(a) Personalizing Firm (b) Non-personalizing Firm

Figure 2 Illustration of the equilibrium solution

As discussed in Proposition 2(b), profits of both the firms increase with ψ_1. Hence, when the data sharing happens, for a given value of σ_1, profits of both the firms are higher when $\psi_1 = 0.04$ than when $\psi_1 = 0.02$.

6 Extensions

We made several assumptions in the base model, such as (i) there's no additional fit cost of the product that the customer purchases from the non-personalizing firm, (ii) the cost of maintaining a recommender system (and improving it) is 0, (iii) the profile quality of all customers are same at a time, and (iv) all customers have same reservation prices. We will relax these assumptions one by one and show that the main results regarding the existence of the data sharing equilibrium (willingly by both firms or induced by the personalizing firm) hold in all these cases. For brevity, we have provided all the expressions of u^*, prices, and profits in the E-Companion, and present only the relevant expressions here. These expressions are derived in the same manner the base model expressions are derived. We only discuss the main insights in this section.

6.1 Substitution Cost

So far we have assumed that when a customer obtains a product from the non-personalizing firm, she does not incur any additional cost. However, it is possible that while purchasing from the the non-personalizing firm, the customer needs to spend effort in browsing through the products and analyzing them in an attempt to find her preferred product. Hence, she may incur an additional search cost while purchasing from the non-personalizing firm. Past studies show that although the search is easier on the internet than in the physical world, search cost does exist. For example, Johnson et al. (2004) empirically show that the customers usually search 1.2 book sites and 1.3 CD sites. Besides, the studies related to price dispersion on internet provide evidence that the customers incur search costs while searching products in e-commerce websites (Walter et al. 2006).

Also, despite the additional search effort, sometimes the customer may not be able to find her preferred product, and may have to choose a substitute (not necessarily a lower quality) product. The fit cost of this substitute may be higher than that of her preferred product. Thus, the customer may incur an additional cost when purchasing from the non-personalizing firm, referred to as the *substitution cost* ($\gamma > 0$), consisting of (i) the search cost, and (ii) the cost of purchasing a substitute. Even if the cost of purchasing the substitute product becomes 0 in expectation (as a result of the customer sometimes finding a better than preferred product at random from the non-personalizing firm), the search cost usually exists, and therefore γ is usually greater than zero.

The new problem is a generalization of the problem we studied. By solving the new objective functions of firms and customer, we find the equilibrium profits as stated in the following proposition.

LEMMA 3. *At period T_2, the equilibrium profits of the personalizing and non-personalizing firms, respectively, are:*

(a) $\Pi_1^{DS^*} = \frac{((\xi(2+B)\alpha-\sigma_1+3\psi_1)+\beta\gamma)^2}{18(\alpha+\psi_1)\beta\xi}$ *and* $\Pi_2^{DS^*} = \frac{(\xi((4-B)\alpha+\sigma_1+3\psi_1)+\beta\gamma)^2}{18(\alpha+\psi_1)\beta\xi}$ *when data is shared.*

(b) $\Pi_1^{NDS^*} = \frac{((2+B)\alpha\xi+\beta\gamma)^2}{18\alpha\beta\xi}$ *and* $\Pi_2^{NDS^*} = \frac{((-4+B)\alpha\xi+\beta\gamma)^2}{18\alpha\beta\xi}$ *when data is not shared.*

The accompanying condition on values of B is provided in the E-Companion. At time T_1, data sharing happens when c_1 (modified for the new context) is satisfied, where $c_1 \coloneqq \sigma_1 < \frac{6\alpha\sigma_1+((2+B)\alpha+\beta\gamma)\left((-4+B)\alpha+\beta\gamma+\sqrt{((-4+B)\alpha+\beta\gamma)^2+12\alpha\sigma_1}\right)}{18\alpha}$.

Our analysis shows that when $\gamma > 0$, the mechanism of data sharing between the two firms is similar to what we have discussed earlier. Also, the comparative statics reveal that the effects of α,

ψ_1, and σ_1, on fractions of purchases, profile quality, and the prices and profits of the firms remain qualitatively the same. Further, the increase in the recommender system effectiveness may increase the profits of both firms. These findings demonstrate that our model and the results are robust.

6.2 Cost of Recommendation

We now consider the possibility of a non-zero cost of maintaining a recommender system. Suppose the personalizing firm incurs a cost c per unit α, the recommender system effectiveness, in maintaining the system. Now, the cost considerations will dictate what effectiveness the personalizing firm offers to the customers. This decision must precede the data sharing decision because regardless of the final outcome of the data sharing game, α must be decided based on the firm's costs. Therefore, the firm decides α at time T_0 based on profile quality maintained using its own data.

We assume that the cost of maintaining the recommender system increases in α at an increasing rate. The α may be decided based on two considerations: (i) the firm behaves like a profit maximizer, and its objective is, $\max_{\alpha} \Pi_1 = \max_{\alpha} \frac{(2+B)^2\alpha\xi}{18\beta} - c\alpha^2$, or (ii) the firm decides to provide quality service to its customers and select α to maximize the profile quality of the customers while balancing it against its cost of maintaining the system, i.e., $\max_{\alpha} \xi x - c\alpha^2 = \max_{\alpha} \frac{\xi\alpha u^*(B-u^*)}{\beta} - c\alpha^2 = \max_{\alpha} \frac{(2+B)(-2+5B)\alpha\xi}{36\beta} - c\alpha^2$. Following proposition presents the α's in both cases.

Proposition 6. *A. When the firm is a profit maximizer, then* $\alpha = \frac{\xi(2+B)^2}{36c\beta}$.
B. When the firm decides to provide quality service to its customers, then $\alpha = \frac{\xi(2(B+2)^2+3B^2-12)}{72c\beta}$.

In both cases, the second order derivative is $-2c < 0$, which means that the corresponding values of α maximize the respective objective functions. It is easy to verify that the α in the second case is larger than the α in the first case (since $B > 2$), which confirms that in the second case the firm provides a better profile quality for the same u. Rest of the results corresponding to the remaining periods are not impacted because α is fixed after this stage.

6.3 Heterogeneity in Profile Quality

So far we considered that the profile quality was homogeneous. However, at one point of time, different customers may have different profile qualities, and the firm should decide prices based on such variations in profiles. In this section, we will verify whether data sharing happens (or not) when profile quality is heterogenous. Specifically, the profile quality is a function of time t, the length of duration a customer has been shopping with the two firms. The customers are assumed to be uniformly distributed in t, where $t \in [\epsilon, T]$, ϵ is close to 0 and T is the maximum time any customer has been actively purchasing from the two firms.

Intuitively, all the scenarios as in Proposition 5 should exist because heterogenous profile quality is just a generalization of homogeneous profile quality. The expressions of all important decision variables are provided in the E-Companion. The Lemma below shows the profits of the firms at time T_2.

LEMMA 4. *At period T_2, the equilibrium profits of the personalizing and non-personalizing firms, respectively, are:*

(i) $\Pi_1^* = \Pi_1^{DS^*} = \frac{(T-\epsilon)\xi}{ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}} \frac{((2+B)\alpha-\sigma_1+3\psi_1)^2}{18\beta(\alpha+\psi_1)}$ *and* $\Pi_2^* = \Pi_2^{DS^*} = \frac{(T-\epsilon)\xi}{ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}} \frac{((4-B)\alpha+\sigma_1+3\psi_1)^2}{18\beta(\alpha+\psi_1)}$ *when data is shared.*

(ii) $\Pi_1^* = \Pi_1^{NDS^*} = \frac{(T-\epsilon)\xi}{ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}} \frac{(2+B)^2\alpha}{18\beta}$ *and* $\Pi_2^* = \Pi_2^{NDS^*} = \frac{(T-\epsilon)\xi}{ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}} \frac{(4-B)^2\alpha}{18\beta}$ *when data is not shared.*

Note that all the profit expressions are scaled by the factor $\frac{(T-\epsilon)\xi}{ln\frac{(1-e^{-T\beta})}{(1-e^{-\epsilon\beta})}}$ when compared to the corresponding expressions in Lemma 2. The prices are also scaled by constants (see E-Companion). Since these scaling constants do not affect the comparative statics results, our analyses in Section 4 for prices and profits remain valid. Moreover, since the profits of both firms are scaled by the same constant, the data sharing conditions as in Proposition 5 also continue to hold. Hence, considering heterogeneity in the profile quality bears no impact on any of the analyses.

6.4 Heterogeneity in Reservation Price

We assumed in our main model that the reservations prices of all customers are the same and it is sufficiently high so that every customer purchases products. In this section, we relax that assumption and show that the data sharing can happen (firms participate willingly, or the personalizing firm can induce data sharing). We assume that the reservation price R is uniformly distributed as

$R \in [R_0, R_1]$, so that customers with $R = R_0$ have negative surpluses and customers with $R = R_1$ have positive surpluses, respectively, if they purchase products. The problem of finding the equilibrium is analytically intractable, and the relevant expressions are provided in the E-Companion. However, it is possible to construct a numerical example to show the existence of a data sharing equilibrium as presented in Proposition 5. For instance, if $R_0 = 2$, $R_1 = 5$, $A = 3$, $\alpha = 0.2$, $B = 2.1$, $\beta = 1$, $\xi = 1$ $\psi_1 = 0.12$, and $\sigma_1 = 0.012$, then the two firms willingly share data with each other. In this case, $u^* = 0.627$, and the profits of the personalizing and the non-personalizing firms are 0.425 and 0.162, respectively. When there is no data sharing, $u^* = 0.702$, the profits of the two firms are 0.356 and 0.069, respectively, i.e., lower than when firms participate in data sharing alliance. Note that u^* is same for all the customers regardless of the reservation price.

7 Conclusions

Recommender systems have contributed significantly to the fast growth of e-commerce firms, and data is central to the operations of these systems used by personalizing firms. It is important for these firms to improve the profile qualities of customers so that the customers can obtain products that they prefer. Better profiles leading to better recommendations create additional value for customers, which enables the firm to increase demand and/or price and profit. Research on recommender system design shows that additional data can lead to improvements in profile qualities of users. We investigate how a personalizing firm can use this important insight for improving customer profiles to increase its profit.

Additional data affects the profile quality in two ways: it increases rate at which the recommender system learns the profile (learning rate influence), and it improves the profile qualities of individual customers (an across the board improvement, called profile influence). We focus on a situation where a personalizing firm competes with a non-personalizing firm (that does not provide any recommendations) and the two firms may enter into a data sharing alliance if that benefits both of them. In this arrangement, the non-personalizing firm transfers its data to the personalizing firm. The customers are strategic as they first find products they want to purchase by using the recommendations of the personalizing firm (the preferred products), and then they may purchase the products from either of the firm. For a customer, purchasing from the personalizing firm maintains a certain level of profile quality that she uses to obtain good recommendations (better profile quality helps in finding products that the customer prefers and eventually increases her surplus), and purchasing from the non-personalizing firm also increases her surplus because typically the prices of the products are lower as compared to the personalizing firm. The important question we investigate is, when can such an alliance exist? What incentive a non-personalizing firm has in participating in such an alliance? To answer that question, the model is setup as a two stage game. In the first period, T_1, the firms decide whether to share data. In the second period, T_2, firms play a simultaneous move price game when customers purchase products from both the firms. We derive the subgame perfect Nash equilibrium.

We find that the personalizing firm may benefit from using additional data shared by the non-personalizing firm under certain conditions. The non-personalizing firm, on the other hand, always benefits from such an alliance. Thus, it is possible that both firms are willing to participate in the alliance, which happens when the learning rate influence on the profile quality is usually more than the profile quality influence. An implication of this result for the personalizing firm is that it should engage with the non-personalizing firm to facilitate the higher learning rate improvement from using the shared data. We find that although data sharing can lead to lower customer surplus, it can also increase the social surplus.

These findings have major implications for both firms. Data sharing is an alternative approach for the personalizing firm to improve profile qualities, other than improving the recommender system design (e.g., the algorithm). This is especially useful if for some reason, improving the system's effectiveness becomes incredibly expensive for the personalizing firm (e.g., technological or financial barriers exist in improving the algorithm further). Further, this data sharing alliance creates an opportunity for the non-personalizing firm to monetize from its data, which otherwise is nothing but an inventory with no use that directly adds to the firm's bottom line. Importantly, this is the first study of its kind that uses the insights developed in the literature of recommender system design and use them in an economic setting to show the existence of the possibility of an ecosystem where competing firms can benefit each other.

These results have broad implications on policies related to facilitating customer information and data sharing between firms. Given that data sharing between competitors can improve social surplus, regulators may want to create an environment where firms can share data with each other. Critics often argue that firms exploit customers by collecting data and charging higher prices. Instances of data sharing between competitors can only add fuel to this fire. However, experts are now arguing in favor of data collection because the benefits of better services using data mostly outweigh the costs (Thierer 2013, Kelly 2017). Our research should provide support to this argument as we show that firms can generate substantial social surplus by providing better services through additional data.

We also find that when the personalizing firm improves its recommender system (essentially by improving the system's design), not only the profit of that firm increases, but the profit of the non-personalizing firm increases as well. However, the profit of the non-personalizing firm may also decrease if the personalizing firm uses the shared data and then it improves its system. Therefore, the non-personalizing firm may have to consider any possibilities of improvement of the recommender system effectiveness of the personalizing firm in the future after it shares its data with the personalizing firm. When the non-personalizing firm shares its data and the personalizing firm improves its system, the demand of the personalizing firm increases. This can cause a drop in the inventory of the products with high demand (Demirezen and Kumar 2017). The personalizing firm may have to adjust its recommendations by considering the possibilities of lost opportunities because of no existing inventory for products, triggered by the increased demand.

We show that increased learning rate (which improves the recommendation system) due to data sharing increases the differentiation between the firms. Thus, both firms benefit when the learning rate of the personalizing firm increases due to an increase in the learning rate influence of the shared data. Although an increased learning rate causes a decrease in the purchases from the personalizing firm, the loss is offset as the firm increases its price. The non-personalizing firm also increases its price. When the profile influence of the shared data increases, profile quality for customers improve even when they do not purchase from the personalizing firm. As a result, customers purchase less from the personalizing firm. To dampen the demand reduction, the personalizing firm reduces its price, and consequently its profit reduces. Increased demand of the non-personalizing firm increases its profit (also, the non-personalizing firm increases its price).

This research is a first step in an attempt to analyze the effect of data sharing on firms in a duopoly competition setting where only one firm provides personalizing services, and customers purchase from both firms. Further research may consider multiple firms in a similar setting. It may be interesting to study how data sharing may happen between two personalizing firms, although the strategic behavior of the customers may not be meaningful in that context because the customer may want to check out the recommendations of the competing firm rather than purchase the product recommended by our personalizing firm.

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Electronic Companion: Notation and Proofs

EC.1 Notation

The following table describes the notation used in the main paper.

SYMBOL	DEFINITION	REMARKS
u	Fraction of purchases from the personalizing firm	**Decision Variable** for the customer
p_1	Price charged by the personalizing firm	**Decision Variable** for the personalizing firm
p_2	Price charged by the non-personalizing firm	**Decision Variable** for the non-personalizing firm
k_1	Binary sharing variable of the personalizing firm	**Decision Variable** for the personalizing firm
k_2	Binary sharing variable of the non-personalizing firm	**Decision Variable** for the non-personalizing firm
x	Profile quality	
A	Maximum fit cost	
B	profile augmentation constant	Higher B increases the rate of profile improvement
R	Reservation price of the customer	$R > 0$
S	Total surplus of the customer	
α	Effectiveness of the recommender system	A higher value indicates that the profile quality improves faster for the same level of u
η	Learning rate influence variable as a result of data sharing	
δ	Profile influence variable as a result of data sharing	
ψ_1	Value of Learning rate influence as a result of data sharing	
σ_1	Value of Profile influence as a result of data sharing	

Table EC.1 Parameters and Variables

EC.2 Proofs of Propositions and Lemmas

In this section we provide the proofs of the propositions and the lemmas in the main paper.

Proof of Lemma 1: First, we solve the equation $\dot{x} = (\alpha(B-u) + \eta(1-u))u + \delta(1-u) - \beta x$. This is an ODE of first order and has the solution $x(t) = (1-e^{-\beta t})\frac{Bu\alpha+\delta-u(\delta-\eta+u(\alpha+\eta))}{\beta}$. For our analysis, we are interested in the long term stable solution. Thus, using $x = \lim_{t\to\infty}(1-e^{-\beta t})\dfrac{Bu\alpha+\delta-u(\delta-\eta+u(\alpha+\eta))}{\beta}$, we get

$$x = \frac{Bu\alpha+\delta-u(\delta-\eta+u(\alpha+\eta))}{\beta} \tag{EC.1}$$

We substitute this expression of x in $S = R - u(A - \xi x + p_1) - (1-u)(A - \xi x + p_2)$. The resulting expression, as shown below, is continuous and differentiable in u.

$$S = -A - \mathrm{q}2 + R + \frac{\delta\xi}{\beta} + u^2\left(-\frac{\alpha\xi}{\beta} - \frac{\eta\xi}{\beta}\right) + u\left(-\mathrm{q}1 + \mathrm{q}2 + \frac{B\alpha\xi}{\beta} - \frac{\delta\xi}{\beta} + \frac{\eta\xi}{\beta}\right)$$

Differentiating it with respect to u and solving the first order equation provides $u = \frac{-\beta p_1+\beta p_2+B\alpha-\delta+\eta}{2(\alpha+\eta)}$. The second order derivative is $-2\xi\left(\frac{\alpha+\eta}{\beta}\right) < 0$ because $\alpha > |\eta|$, which means that the first order solution is a maxima.

Data sharing happens means $k_1k_2 = 1$. The expressions for u can be obtained by substituting $\eta = \psi_1$ and $\delta = \sigma_1$. When data sharing does not happen $k_1k_2 = 0$, i.e., $\eta = \sigma = 0$ and using these we can find the expression of u when data is not shared. Using $0 < u < 1$ for the two cases, we can find the corresponding ranges of B. ■

Proof of Lemma 2: When data is shared ($k_1k_2 = 1$, $\eta = k_1k_2\psi_1$, and $\delta = k_1k_2\sigma_1$). The expressions for the other two cases can be found the same way.

$$\Pi_1 = up_1, \text{ and } \Pi_2 = (1-u)p_2,$$

where u is used from Lemma 1. Thus,

$$\frac{d\Pi_1}{dp_1} = \frac{\xi(\alpha B - \sigma_1 + \psi_1) - 2p_1\beta + p_2\beta}{2\xi(\alpha+\psi_1)}, \text{ and } \frac{d\Pi_2}{dp_2} = \frac{\xi(2\alpha - \alpha B + \sigma_1 + \psi_1) + p_1\beta - 2p_2\beta}{2\xi(\alpha+\psi_1)}.$$

The second order derivatives of the profits with respect to prices are negative, indicating that the prices are profit maximizing. By solving these equations for p_1 and p_2, we get the equilibrium expressions. By substituting them in u, prices, and profit expressions, we get the respective equations shown in Lemma 2(a).

Next, we derive the conditions for maintaining a market with a duopoly. Note that no data sharing should always be a valid option even when data sharing eventually happens. Thus, we first establish the condition for B when no data is shared. We know that $B > 2$ from Section 3.2 and from $u^* < 1$ we have $B < 4$. Thus, $2 < B < 4$.

When data is shared, using $0 < u^* < 1$, we obtain the following conditions:

$$B > \frac{\sigma_1 - 3\psi_1}{\alpha} - 2, \text{ and} \tag{EC.2}$$

$$B < 4 + \frac{3\psi_1 + \sigma_1}{2}. \tag{EC.3}$$

Combining these conditions with Equation (4), we get what is shown in the Lemma 2(a) for B. Note that Condition (EC.3) is already satisfied because we assume $B < 4$. ■

Proof of Proposition 1: (a) From Lemma 2(a), we get $\frac{du^*}{d\alpha} = \frac{\sigma_1 + (-1+B)\psi_1}{6(\alpha+\psi_1)^2} > 0$. From Equation (EC.1) and expression on u^* in Lemma 2(a), we find the equilibrium profile quality. Differentiating it with α we get: $\frac{dx^*}{d\alpha} = \frac{\alpha^2(B+2)(5B-2)+2\alpha(B+2)(5B-2)\psi_1-10(B-1)\sigma_1\psi_1+9(2B-1)\psi_1^2-5\sigma_1^2}{36\beta(\alpha+\psi_1)^2} = \frac{(8B-4)\alpha^2+5(B^2\alpha^2-\sigma_1^2)+9(2B-1)+(16B-8)\alpha\psi_1+10B\psi_1(B\alpha-\sigma_1)+10\psi_1\sigma_1}{36\beta(\alpha+\psi_1)^2}$

Using $B > 2$ and $B > 1 + \frac{\sigma_1}{\alpha}$ (which implies $B\alpha - \sigma_1 > \alpha > 0$) from $B > 2 + \frac{\sigma_1+\psi_1}{\alpha}$, $\frac{dx^*}{d\alpha} > 0$.

$\frac{dp_1^*}{d\alpha} = \frac{\xi(2+B)}{3\beta} > 0$. $\frac{dp_2^*}{d\alpha} = \frac{\xi(4-B)}{3\beta} > 0$ using $B > 4$ from boundary condition in Lemma 2(a).

We now analyze the profit. We know that $\frac{d\Pi_1^*}{d\alpha} = \frac{\xi(\alpha(B+2)-\sigma_1+3\psi_1)(\alpha(B+2)+2B\psi_1+\sigma_1+\psi_1)}{18\beta(\alpha+\psi_1)^2}$. Using boundary condition in Lemma 2(a) we get $(\alpha(B+2) - \sigma_1 + 3\psi_1) > 0$. Thus, $\frac{d\Pi_1^*}{d\alpha} > 0$.

Next, we derive the condition when $\frac{d\Pi_2^*}{d\alpha} > 0$. We know that $\frac{d\Pi_2^*}{d\alpha} = \frac{\xi(\alpha(4-B)+\sigma_1+3\psi_1)(\alpha(4-B)-(2B-5)\psi_1-\sigma_1)}{18\beta(\alpha+\psi_1)^2}$. Then, $(\alpha(4-B) + \sigma_1 + 3\psi_1) > 0$ by Lemma 2(a). Π_2^* increases with an increase in α when $\frac{d\Pi_2^*}{d\alpha} > 0$, i.e., $(\alpha(4-B) - (2B-5)\psi_1 - \sigma_1) > 0$, which is the condition provided in the proposition.

(b) $\frac{du^*}{d\alpha}=0$, $\frac{dx^*}{d\alpha}=\frac{(B+2)(5B-2)}{36\beta}>0$, $\frac{dp_1^*}{d\alpha}=\frac{2+B}{3\beta}>0$, $\frac{dp_2^*}{d\alpha}=\frac{4-B}{3\beta}>0$, $\frac{d\Pi_1^*}{d\alpha}=\frac{(2+B)^2}{18\beta}>0$, and $\frac{d\Pi_2^*}{d\alpha}=\frac{(4-B)^2}{18\beta}>0$ ■

Proof of Proposition 2: Using Lemma 2(a) we find $\frac{du^*}{d\psi_1}=\frac{\alpha-B\alpha+\sigma_1}{6(\alpha+\psi_1)^2}$. From Equation (4) we infer that $B\alpha-\sigma_1-\alpha>0$, which means $\frac{du^*}{d\psi_1}<0$. $\frac{dx^*}{d\psi_1}=\frac{1}{4\beta}-\frac{5(\alpha-B\alpha+\sigma_1)^2}{36\beta(\alpha+\psi_1)^2}$. By solving the equation $\frac{dx^*}{d\psi_1}>0$ for its roots, we can find the condition provided in the proposition for the increase in the profile quality. From the price expressions in Lemma 2(a), we get $\frac{dp_1^*}{d\psi_1}=\frac{dp_2^*}{d\psi_1}=\frac{\xi}{\beta}>0$. Also,

$$\frac{d\Pi_1^*}{d\psi_1}=\frac{d\Pi_2^*}{d\psi_1}=\frac{\xi(\alpha(4-B)+\sigma_1+3\psi_1)(\alpha(B+2)-\sigma_1+3\psi_1)}{18\beta(\alpha+\psi_1)^2}.$$

Using boundary conditions in Lemma 2(a), we find that both the profit derivatives are positive. ■

Proof of Proposition 3: From Lemma 2(a), we find that $\frac{du^*}{d\sigma_1}=-\frac{1}{6(\alpha+\psi_1)}<0$. $\frac{dx^*}{d\sigma_1}=\frac{\alpha(14-5B)+5\sigma_1+9\psi_1}{18\beta(\alpha+\psi_1)}$. By solving the equation $\frac{dx^*}{d\sigma_1}>0$, we can find the condition provided in the proposition for when the profile quality increases. From price expressions in Lemma 2(a), we get $\frac{dp_1^*}{d\sigma_1}=\frac{-\xi}{3\beta}<0$, and $\frac{dp_2^*}{d\sigma_1}=\frac{\xi}{3\beta}>0$. Finally,

$$\frac{d\Pi_1^*}{d\sigma_1}=\frac{-\xi(\alpha(B+2)-\sigma_1+3\psi_1)}{9\beta(\alpha+\psi_1)}<0,\text{ and }\frac{d\Pi_2^*}{d\sigma_1}=\frac{\xi(\alpha(4-B)+\sigma_1+3\psi_1)}{9\beta(\alpha+\psi_1)}>0.$$

The last two inequalities are due to the boundary conditions shown in Lemma 2(a). ■

Proof of Proposition 4

$$\begin{aligned}\Delta x=&x(\psi=\psi_1,\sigma=\sigma_1)-x(\psi=0,\sigma=0)\\=&5\sigma_1^2+2\sigma_1\left((14-5B)\alpha+9\psi_1\right)+\psi_1\left((4-5(-2+B)B)\alpha+9\psi_1\right)\end{aligned}$$

The profile quality of a customer improves when shared data is strongly compatible iff $\Delta x>0$. We get the requisite conditions by solving these expressions for σ_1. ■

Proof of Proposition 5: When there is no data sharing, $k_1=k_2=0$. The profits are:

$$\Pi_1^{NDS^*}=\frac{1}{18\beta}\xi\alpha(B+2)^2,\text{ and }\Pi_2^{NDS^*}=\frac{1}{18\beta}\xi\alpha(4-B)^2. \tag{EC.4}$$

When data is shared, the profits are:

$$\Pi_1^{DS^*}=\frac{\xi(\alpha(B+2)-\sigma_1+3\psi_1)^2}{18\beta(\alpha+\psi_1)},\ \Pi_2^{DS^*}=\frac{\xi(\alpha(4-B)+\sigma_1+3\psi_1)^2}{18\beta(\alpha+\psi_1)} \tag{EC.5}$$

Here, $p_1^* > p_2^*$ because $p_1^* - p_2^* = \frac{2\xi((-1+B)\alpha - \sigma_1)}{3\beta} > 0$ from Equation (4). Next, we find the condition when the personalizing firm's profit increases. Condition c_1 ensures that the profit of the personalizing firm is higher when data is strongly compatible; we use $\Pi_1^{DS^*} > \Pi_1^{NDS^*}$ to obtain the condition.

$$\Pi_1^{DS^*} - \Pi_1^{NDS^*} = \frac{\xi(\sigma_1^2 - 2\sigma_1\left((2+B)\alpha + 3\psi_1\right) + \psi_1\left(-(-4+B)(2+B)\alpha + 9\psi_1\right))}{18\beta\left(\alpha+\psi_1\right)}. \tag{EC.6}$$

$\Pi_1^{DS^*} - \Pi_1^{NDS^*} > 0$ from Equation (EC.6) has roots of form $\chi_1 = X_1 + X_2$ and $\chi_2 = X_1 - X_2$ where $X_1 = \frac{1}{18}\left(6\sigma_1 + (2+B)(-4+B)\alpha\right)$ and $X_2 = \frac{1}{18}\left((2+B)\sqrt{\alpha}\sqrt{(-4+B)^2\alpha + 12\sigma_1}\right)$. Thus, $\psi_1 > \chi_1$ ensures the required condition of $\Pi_1^{DS^*} - \Pi_1^{NDS^*} > 0$. The boundary conditions are satisfied here.

Now, we establish that when the data is shared, the non-personalizing firm has always a higher profit than when data is not shared. Here,

$$\Pi_2^{DS^*} - \Pi_2^{NDS^*} = \frac{\xi((\sigma_1 + 3\psi_1)^2 + (4-B)\,\alpha\left(2\sigma_1 + 2\psi_1 + B\psi_1\right))}{18\beta(\alpha+\psi_1)} \geq 0. \tag{EC.7}$$

The conclusion above is because of $B < 4$. It is now easy to conclude that when c_1 satisfies, both firms are willing to share data, i.e., $k_1 = k_2 = 1$. Otherwise, $k_2 = 1$ but $k_1 = 0$, and data sharing does not happen. ■

Expressions for the Extensions

Expressions for 6.1 The objective function of the customer to solve with data sharing variables is

$$\begin{aligned}
&\max_u \left(R - u\left(A - \xi x + p_1\right) - (1-u)\left(A - \xi x + p_2 + \gamma\right)\right),\\
&\dot{x} = \left(\alpha\left(B-u\right) + \eta\left(1-u\right)\right)u + \delta\left(1-u\right) - \beta x,\\
&0 \leq u \leq 1.
\end{aligned} \tag{EC.8}$$

Following are the Nash Equilibrium purchase fractions from the personalizing firm, prices, and profits when

1. data sharing happens,

   (i) $u^* = \frac{(2+B)\alpha - \sigma_1 + 3\psi_1 + \beta\gamma}{6(\alpha+\psi_1)}$.

(ii) $p_1^* = \frac{\xi((2+B)\alpha-\sigma_1+3\psi_1)+\beta\gamma}{3\beta}$, and $p_2^* = \frac{\xi(4\alpha-B\alpha+\sigma_1+3\psi_1)-\beta\gamma}{3\beta}$.

(iii) $\Pi_1^* = \Pi_1^{DS^*} = \frac{(\xi((2+B)\alpha-\sigma_1+3\psi_1)+\beta\gamma)^2}{18\beta(\alpha+\psi_1)}$, and $\Pi_2^* = \Pi_2^{DS^*} = \frac{(\xi((4-B)\alpha+\sigma_1+3\psi_1)+\beta\gamma)^2}{18\beta(\alpha+\psi_1)}$.

Market with duopoly exist when $\max\left\{\left(\frac{\sigma_1-3\psi_1-\beta\gamma}{\alpha}-2\right),\left(2+\frac{\psi_1+\sigma_1}{\alpha}\right)\right\} < B < 4$.

2. data sharing does not happen,

(i) $u^* = \frac{(2+B)\alpha+\beta\gamma}{6\alpha}$.

(ii) $p_1^* = \frac{(2+B)\alpha\xi+\beta\gamma}{3\beta}$, and $p_2^* = \frac{\xi(4\alpha-B\alpha)-\beta\gamma}{3\beta}$.

(iii) $\Pi_1^* = \Pi_1^{NDS^*} = \frac{(2+B)^2\alpha\xi+\beta\gamma}{18\beta}$, and $\Pi_2^* = \Pi_2^{NDS^*} = \frac{(4-B)^2\alpha\xi-\beta\gamma}{18\beta}$.

Market with duopoly exist when $2 < B < 4 - \frac{\beta\gamma}{\alpha}$.

Expressions for 6.3 The profile quality as a function of t is $x(t) = (1-e^{-\beta t})\frac{Bu\alpha+\delta-u(\delta-\eta+u(\alpha+\eta))}{\beta}$. Using that we find u that maximizes the surplus $S(t)$ as $u(t) = \frac{1}{2}\left(B - \frac{(p_1-p_2)\beta}{\alpha(1-e^{-t\beta})}\right)$. The profit equations for the personalizing firm are $\Pi_1 = \int_\epsilon^T up_1dt$ and $\Pi_2 = \int_\epsilon^T(1-u)p_2dt$.

Following are the Nash Equilibrium purchase fractions from the personalizing firm, prices, and profits when

1. data sharing happens,

(i)

$$u^* = \frac{\begin{pmatrix} -2(-1+B)e^{t\beta}\alpha\beta(T-\epsilon)+3B\left(-1+e^{t\beta}\right)\alpha\left(ln\left(1-e^{T\beta}\right)-ln\left(1-e^{\beta\epsilon}\right)\right) \\ +\left(2e^{t\beta}\beta(T-\epsilon)-3\left(-1+e^{t\beta}\right)ln\left(1-e^{T\beta}\right)+3\left(-1+e^{t\beta}\right)ln\left(1-e^{\beta\epsilon}\right)\right) \\ \sigma_1+3\left(-1+e^{t\beta}\right)\left(ln\left(1-e^{T\beta}\right)-ln\left(1-e^{\beta\epsilon}\right)\right)\psi_1 \end{pmatrix}}{6\left(-1+e^{t\beta}\right)\left(ln\left(1-e^{T\beta}\right)-ln\left(1-e^{\beta\epsilon}\right)\right)(\alpha+\psi_1)}$$

(ii) $p_1^* = \frac{1}{ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}}\frac{\xi(2\alpha+B\alpha-\sigma_1+3\psi_1)}{3\beta}$ and $p_2^* = -\frac{1}{ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}}\frac{\xi(-4\alpha+B\alpha-\sigma_1-3\psi_1)}{3\beta}$

(iii) $\Pi_1^* = \Pi_1^{DS^*} = \frac{(T-\epsilon)}{ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}}\frac{\xi((2+B)\alpha-\sigma_1+3\psi_1)^2}{18\beta(\alpha+\psi_1)}$ and $\Pi_2^* = \Pi_2^{DS^*} = \frac{(T-\epsilon)}{ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}}\frac{\xi((4-B)\alpha+\sigma_1+3\psi_1)^2}{18\beta(\alpha+\psi_1)}$.

2. data sharing does not happen,

(i) $u^* = \frac{B}{2} - \frac{(-1+B)}{3\left(1-e^{-t\beta}\right)\left(ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}\right)}$,

(ii) $p_1^* = \frac{\xi(2+B)\alpha}{3\beta\left(ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}\right)}$ and $p_2^* = \frac{\xi(4-B)\alpha}{3\beta\left(ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}\right)}$,

(iii) $\Pi_1^* = \Pi_1^{NDS^*} = \frac{(T-\epsilon)}{ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}}\frac{\xi(2+B)^2\alpha}{18\beta}$ and $\Pi_2^* = \Pi_2^{NDS^*} = \frac{(T-\epsilon)}{ln\frac{1-e^{-T\beta}}{1-e^{-\epsilon\beta}}}\frac{\xi(4-B)^2\alpha}{18\beta}$.

Expressions for 6.4 Using the expression of u for a particular case, data shared and data not shared, we find for respective cases the R of the customers who purchase using the condition $S \geq 0$. The reservation price of a customer who purchases from the two firms is $R \geq (A-\xi x)+up_1+(1-u)p_2 = r$ where u and x are

1. $u = \frac{B\alpha - p_1\beta + p_2\beta}{2\alpha}$ and $x = \frac{Bu\alpha - u^2\alpha}{\beta}$ when no data is shared, and
2. $u = \frac{B\alpha - p_1\beta + p_2\beta - \sigma_1 + \psi_1}{2(\alpha+\psi_1)}$ and $x = \frac{Bu\alpha + \sigma_1 - u(\sigma_1 - \psi_1 + u(\alpha+\psi_1))}{\beta}$ when data is shared.

The profit equations for the personalizing and non-personalizing firms are $\Pi_1 = \int_r^{R_1} up_1 dt$ and $\Pi_2 = \int_r^{R_1}(1-u)p_2 dt$. When these equations are used to determine the Nash Equilibrium, following equations are obtained whose solution for prices will provide equilibrium prices (the expressions are provided in terms of η and δ and by substituting appropriate values we can get the exact conditions for the prices for the respective cases).

$$-\frac{1}{8(\alpha+\eta)^2\xi^2}\left(4\beta^2p_1^3 - 9\beta p_1^2\left((B\alpha-\delta+\eta)\xi+\beta p_2\right)+2p_1\left(\xi\left(-4(A-R)\beta(\alpha+\eta)+\right.\right.\right.$$
$$\left.\left(3B^2\alpha^2+4\alpha\delta+3\delta^2-2\delta\eta+3\eta^2+6B\alpha(-\delta+\eta)\right)\xi\right)+2\beta((-2+3B)\alpha-3\delta+\eta)\xi p_2+3\beta^2p_2^2\Big)$$
$$-\frac{1}{\beta}\left((B\alpha-\delta+\eta)\xi+\beta p_2\right)\left(\left(B^2\alpha^2+4\alpha\delta+2B\alpha(-\delta+\eta)+(\delta+\eta)^2\right)\xi^2+\beta^2p_2^2-2\beta\xi\left(2A(\alpha+\eta)\right.\right.$$
$$\left.\left.\left.-2R(\alpha+\eta)+(2\alpha-B\alpha+\delta+\eta)p_2\right)\right)\right)=0, \text{ and}$$

$$\frac{1}{8(\alpha+\eta)^2\xi^2} - 2p_2\left((2\alpha-B\alpha+\delta+\eta)\xi+\beta p_1-\beta p_2\right)^2 - p_2\left(\left(B^2\alpha^2+4\alpha\delta+2B\alpha(-\delta+\eta)\right.\right.$$
$$\left.+(\delta+\eta)^2\right)\xi^2+\beta^2\left(p_1-p_2\right)^2-2\beta\xi\left(-2R\alpha+2A(\alpha+\eta)-\delta p_1+B\alpha\left(p_1-p_2\right)+2\alpha p_2+\delta p_2\right.$$
$$\left.\left.+\eta\left(-2R+p_1+p_2\right)\right)\right)+\frac{1}{\beta}\left((2\alpha-B\alpha+\delta+\eta)\xi+\beta p_1-\beta p_2\right)\left(\left(B^2\alpha^2+4\alpha\delta+2B\alpha(-\delta+\eta)\right.\right.$$
$$\left.+(\delta+\eta)^2\right)\xi^2+\beta^2\left(p_1-p_2\right)^2-2\beta\xi\left(-2R\alpha+2A(\alpha+\eta)-\delta p_1+B\alpha\left(p_1-p_2\right)+2\alpha p_2+\delta p_2+\right.$$
$$\left.\left.\eta\left(-2R+p_1+p_2\right)\right)\right)=0$$

The equations are sixth degree polynomials in p_1 and p_2 that cannot be solved simultaneously. Thus, we find numerical solutions for the problem using these equations.